SEDIMENTOLOGY

The exceptional sediment load of fine-grained dispersal systems: Example of the Yellow River, China

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Sedimentary dispersal systems with fine-grained beds are common, yet the physics of sediment movement within them remains poorly constrained. We analyze sediment transport data for the best-documented, fine-grained river worldwide, the Huanghe (Yellow River) of China, where sediment flux is underpredicted by an order of magnitude according to well-accepted sediment transport relations. Our theoretical framework, bolstered by field observations, demonstrates that the Huanghe tends toward upper-stage plane bed, yielding minimal form drag, thus markedly enhancing sediment transport efficiency. We present a sediment transport formulation applicable to all river systems with silt to coarse-sand beds. This formulation demonstrates a remarkably sensitive dependence on grain size within a certain narrow range and therefore has special relevance to silt-sand fluvial systems, particularly those affected by dams.

INTRODUCTION

Sedimentary environments that have fine-grained beds consisting of silt and very fine sand (median grain size range, 15 to 150 μm) are common on Earth and extraplanetary surfaces, with examples including fluvial, deltaic, coastal, and marine settings, as well as subglacial transport systems (Fig. 1A) (1–7). These settings are morphologically active (8), and many have significant relevance to human well-being. For example, low-lying deltaic coastal regions, threatened by ocean storms and rising sea levels, provide living space for approximately 40% of the world’s population (9, 10). To bolster the predictive capabilities of models that assess the evolution and vulnerability of these regions, it is necessary to evaluate the physics of fine-grained dispersal systems (FGDS). However, to date, the mechanics of fine-grained sediment transport remain poorly constrained because of a paucity of field-scale data necessary to properly inform predictive algorithms. In turn, this is due to the difficulty in measuring quasi-equilibrium transport conditions for FGDS, whereby the observational window required to establish approximately steady and uniform flow is rarely achieved. Although flume experiments using fine-grained sediment have yielded sediment transport data (11–16), the formulations developed from these studies have proven inadequate for predictions at field-scale without significant modifications (17–19). Therefore, no physical formulae that claim the ability to predict sediment transport for FGDS show convincing and consistent agreement with both flume and field data (18). In contrast, coarser sediment dispersal systems (for example, sand to gravel) can be characterized by numerous physically based and properly validated sediment transport relations applicable to a range of environments (Fig. 1A).

The lower Huanghe (Yellow River) of China (fig. S1) presents an exceptional opportunity to develop a physics-based sediment transport model applicable to FGDS. It is the most robustly measured fine-grained river system in the world (20), and the spatiotemporal resolution of the measurements is adequate to document quasi-equilibrium sediment transport conditions. We use this rich database from the Huanghe to inform a new model of fine-grained sediment transport that, because it is grounded in basic physics, is applicable to other dispersal systems as well. There are cultural and engineering perspectives underlining the value of using the lower Huanghe to develop a sediment transport formula: This region is the “cradle” of Chinese civilization but is also known as the “sorrow” of the Chinese people because of rapid channel platform change produced by an exceptionally high sediment load (21, 22). The Huanghe traverses 800 km of the North China Plain and, until recently, delivered approximately 1 billion tons/year of sediment to the Bohai Sea with as little as 49 billion tons of water. The river has maintained one of the largest sediment loads worldwide, with a volumetric sediment concentration exceeding large lowland rivers by an order of magnitude (23–25). This sediment, derived from the denudation of friable Loess Plateau soil, has constructed over 30,000 km² of land on the North China Plain over the past several millennia (fig. S1) (26). The river and floodplain system progressively builds the edge of societal sustainability. Seven major avulsions in the past 2000 years have each disrupted 8 to 14 million people, collectively inundating approximately 250,000 km² of floodplain (27). In the past several centuries, there have been greater than 1000 levee breaches (28), and the recurrence interval of channel avulsions is 7 to 14 years (29). Over the past century, sediment deposited within the heavily diked plain has produced riverbed superelevation of as much as 10 m, jeopardizing the 88 million inhabitants living on the floodplain. Over the past several decades, Chinese engineers have stabilized the river by building levees and dikes to constrain channel migration (20). Meanwhile, the Xiaolangdi Dam and Reservoir (fig. S1), which was finished in 2002, stores sediment derived from the Loess Plateau while releasing sediment-depleted water into the lower Huanghe in an effort to reduce bed elevation. These water-release events have occurred annually for the past 15 years as part of the government’s Water and Sediment Regulation Season (WSRS). The effectiveness and impact of Xiaolangdi Dam and WSRS in sustaining the lower Huanghe are under quantitative evaluation (30–32). Developing and implementing engineering measures to ensure future safety of neighboring inhabitants render critical the ability to accurately model sediment and water flux.

Here, we first show that the sediment loads in the Huanghe, based on 10 years of measurements, are an order of magnitude higher than is otherwise predicted by the well-accepted sediment transport formula. Second, this rich database is compiled and used to inform the development of a physically based theory, which, in turn, is validated with field surveys that measure the geometry of the channel bedforms. These efforts of combining a new theoretical framework and field observations yield the primary finding that the Huanghe tends toward upper-regime plane bed,
particularly salient for systems impacted by dams. For example, Brownlie and Brooks (17) collected ~1000 sets of field and flume data, tested 13 sediment transport formulæ, and determined that EH performed best. EH derives from basic physics (33); its dimensionless form reads $C_t q^* = a_{EH} \tau^*^{5/2}$, where $q^* = q_s / \sqrt{RgD^2}$ is the dimensionless sediment discharge, that is, the Einstein number [$q_s = HUC$ is the sediment discharge per unit width ($H$ is the water depth; $U$ is the average flow velocity; and $C$ is the depth-flux-averaged volumetric sediment concentration); $R$ is the sediment submerged specific gravity (1.65 for quartz); $g$ is the gravitational acceleration; and $D$ is the characteristic (nominal) diameter of bed sediment; $C_t = gHS/U^2$ is the bed resistance coefficient ($S$ is the channel bed slope); $\tau^* = HS/(RD)$ is the dimensionless bed shear stress under normal flow conditions, that is, the Shields number; and $a_{EH} = 0.05$. See Materials and Methods concerning these definitions.

The bed material of the lower Huanghe consists of silt and very fine sand, with a median grain diameter ($D_{50}$) of ~70 $\mu$m (Fig. 1A). This is different from most lowland rivers, where median grain size usually exceeds 200 $\mu$m (34). The bed grain size of the lower Huanghe is far below the grain size range that existing formulæ consider (Fig. 1A and table S1). Figure 2A shows that EH underpredicts sediment transport for the Huanghe by an order of magnitude. Nevertheless, because the EH formula is grounded in physics, it is possible to modify and adapt this relation to fine-grained sediment transport systems.

To do so, we use a quasi-equilibrium sediment transport database (see Materials and Methods for the data sources and collection procedures) pertaining to nonhyperconcentrated flow conditions recorded at six hydrological stations located between Huayuankou and Lijin (figs. S1 and S2). Along this stretch of river, the mean grain size distributions (GSDs) of both suspended load and bed material are relatively constant (Fig. 1B). This value varies among rivers systems, although conventionally, the sand-silt transition (that is, 62.5 $\mu$m) is used as a rule of thumb for many sand-bed rivers (39). However, the Huanghe has a fine-grained bed with a median diameter near this value (62.5 $\mu$m). Hence, an alternative value for the wash load/material load cutoff size is inferred based on the standard criterion whereby the cutoff is established as the $D_5$ size (37); using the GSDs of the bed material and suspended sediment from six stations on the lower Huanghe, this value is estimated to be 15 $\mu$m (Fig. 1B). A sensitivity analysis shows that the results shown below are insensitive to the cutoff grain size within a fairly broad range (see Supplementary text).

EH considers energy conservation for a unit fluid column of sediment-laden flow moving along the stoss face of a wavy bedform (for example, a dune) (33). The energy required to elevate the column (per unit time) from trough to crest is $\rho g H \chi U L_{tp}$, where $\rho$ is the water density and $H$ and $L_{tp}$ are the wave height and wavelength, respectively. The driving power is related to the product of the sediment driving force, $\tau_{sf} - \tau_c$, and bed shear velocity, $U^* = \sqrt{\tau_c/\rho}$, where $\tau_{sf}$ is the bed shear stress due to skin friction, $\tau_c$ is the bed shear stress due to bedform form drag, $\tau_c$ is

**THEORY AND RESULTS**

Our starting point is the development of a generalized form of the Engelund-Hansen sediment transport theory (EH) (33), which is a widely validated sand-bed sediment transport relation and therefore provides the basis to compare the Huanghe to other sand-bed rivers. For example, Brownlie and Brooks (17) collected ~1000 sets of field and flume data, tested 13 sediment transport formulæ, and determined that EH performed best. EH derives from basic physics (33); its dimensionless form reads $C_t q^* = a_{EH} \tau^*^{5/2}$, where $q^* = q_s / \sqrt{RgD^2}$ is the dimensionless sediment discharge, that is, the Einstein number [$q_s = HUC$ is the sediment discharge per unit width ($H$ is the water depth; $U$ is the average flow velocity; and $C$ is the depth-flux-averaged volumetric sediment concentration); $R$ is the sediment submerged specific gravity (1.65 for quartz); $g$ is the gravitational acceleration; and $D$ is the characteristic (nominal) diameter of bed sediment; $C_t = gHS/U^2$ is the bed resistance coefficient ($S$ is the channel bed slope); $\tau^* = HS/(RD)$ is the dimensionless bed shear stress under normal flow conditions, that is, the Shields number; and $a_{EH} = 0.05$. See Materials and Methods concerning these definitions.

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the critical shear stress, and \( \tau = \tau_d + \tau_d \). In addition, the total shear stress is \( \tau = \rho g HS = \rho C_d U^2 \) in steady and uniform flow. In regard to the two types of bed stress, skin friction stress is the driving force for sediment transport, whereas form drag stress represents resistance to flow produced by topographic irregularities of the channel and is considered to contribute little to sediment transport. Energy conservation thus dictates \( \rho Rg D \Upsilon_b = e(\tau_d - \tau) U^* \), where \( e \) is an efficiency parameter.

Non-dimensionalizing to the respective Einstein and Shields numbers, \( q'_* = q_*/\sqrt{RgD^3} \) and \( \tau^* = \tau/(\rho g D \Upsilon_b) \), we obtain

\[
C_i q'_* = \frac{C_d L_b}{H_b} \times (\tau^* - \tau_d^*) \sqrt{\tau^*}
\]

EH found that \( C_d L_b/H_b \) takes the constant value 0.235 and that the relation between skin friction shear stress and total shear stress is \( \tau_d^* = 0.4 \tau^* \) for a dune-covered bed. Calibration with flume data yielded \( e = 0.532 \), giving the original EH relation \( C_i q'_* = a_{EH} \tau^* 0.5 \). We hypothesize that the discrepancy between measured and predicted sediment transport (Fig. 2A) could be due to differences in bedform characteristics between the Huanghe and other sand-bed rivers.

EH is based on energy conservation, so the underlying physics can be generalized, whereby the form of the resistance relation (Eqs. 2A and 2B) is used to close Eq. 1

\[
\tau^* - \tau_d^* = b \tau^{n-0.5}
\]

Here, \( \alpha \) is the coefficient, and \( n \) is the exponent of the generalized EH (GEH) formula. Note that \( n \) is also contained in the resistance relation \((n - 0.5)\). Substituting Eqs. 2A and 2B into Eq. 1

\[
C_i q'_* = \alpha \tau^n
\]

The fact that Eqs. 2A and 3 include the same exponent \( n \) underlines the importance of the resistance relation (Eq. 2A) in setting sediment transport capacity \((40)\). Here, both \( \alpha \) and \( n \) are strongly related to the bed state, namely, the presence and geometry of bedforms. A resistance diagram consisting of a plot of \( \tau^* \) versus \( \tau_d^* \) in the original EH exposition shows that when \( n \gtrless 2.0 \), \( n = 1.5 \), and \( n < 1.5 \), the bed state corresponds to dunes, upper plane bed, and antidunes, respectively \((33)\). In Eq. 2B, the aspect ratio \( L_d/H_b \) varies greatly from plane bed to dune regime, whereas \( e, b, C_i \), and \( G_i \) vary less.

The nondimensional numbers \( C_i q'_* \) and \( \tau^* \) yield a good power law relation (similarity collapse) (Fig. 2A), indicating that the GEH accurately describes measured Huanghe sediment flux values; the coefficient and power index values are \( \alpha = 0.90 = 18 \tau_{EH} \) and \( n = 1.68 \), respectively. The difference can be seen in Fig. 2B, wherein we compare the parameters for the Huanghe database with the Guy-Simons-Richardson database \((41)\), which was used to develop the original EH formula.

The coefficient \( \alpha \) and \( n \) show abrupt transitions when the bed material grain size varies through the range of 130 to 190 \( \mu \). Recalling that \( \alpha = \rho C_i L_b/H_b \) and that \( b, C_i \), and \( \tau_{EH} \) are weakly varying coefficients, the large ratio \( \alpha/\tau_{EH} = 18 \) indicates that \( L_d/H_b \) is an order of magnitude larger in the Huanghe than other sand-bed rivers, implying very low amplitude bedform features (dunes). The value \( n = 1.68 \) is close to the value for upper-regime plane bed \( (n = 1.5) \), suggesting the presence
of long-crested bedforms (dunes) approaching upper-regime plane bed in the Huanghe. Thus, form drag can be neglected, with $\tau^*_d \approx 0$ and $\tau^*_f \approx \tau^*$. Both parameters therefore suggest that the phase transition of sediment load described above is associated with change in the state of bed morphology and, specifically, bedform size. This theoretical prediction is verified independently by comparing $\tau^*_f$ with $\tau^*$ (see Supplementary text).

The relative absence of dunes in the lower Huanghe has been previously inferred (42, 43), but direct supporting evidence is sparse. Here, we provide documentation of dune size in the Huanghe using the recently acquired multibeam bathymetry data collected near Lijin during the flood period of 2015 WSRS (Fig. S7) and base-flow period of 2016 (Fig. 3). Prevailing flow conditions facilitated dune development [that is, Froude number (Fr), ~0.23 to 0.5 (44), with the aspect ratio $L_d/H_d$ ranging from 500 to 2000 (1200 for base flow and 500 to 2000 for flood flow). These values are significantly larger than measurements from natural rivers and flume experiments with sand beds, where the aspect
ratio ranges from 10 to 100 \((45, 46)\). As an example, dunes measured in the lower Mississippi River have aspect ratios of 18 to 38, that is, close to those predicted by the standard EH. Therefore, both measurements and predictions indicate that the Huanghe has bedforms (dunes) with aspect ratios that are an order of magnitude larger than most large sand-bed rivers. We speculate that the form of these dunes is due to the fine sand–silt material of the Huanghe bed because this sediment size has a long particle advection length \((47)\) and moves mostly in suspended load, whereas the mixture of bed load and suspended load over a typical sand bed is more conducive to interacting with bedforms that scale with the depth, such as standard dunes \((Fig.\ 2C)\). This point merits further study, and a stability analysis of bedforms on fine-grained beds could potentially provide additional insight.

The present universal GEH formula agrees well with both the lower Huanghe sediment database and the Guy-Simons-Richardson database \((Fig.\ 4)\); notably, the universal GEH formula can predict the dimensionless sediment flux over 10 orders of magnitude, covering characteristic bed material grain size from 15 \(\mu\)m to 1 mm. The performance of the GEH formulation for the fine-grained rivers is also compared to three other relations designed for sediment transport in the Huanghe \(\text{see Supplementary text and table S3). Of the four relations, only the GEH is based on a known value of bed material size; the others require the circularity of prior knowledge about the suspended sediment GSD to predict transport rates.\n
**DISCUSSION AND CONCLUSIONS**

The exceptionally high sediment transport rate measured for the lower Huanghe has a physical basis: Bedforms with large aspect ratios reduce bed material fining via the inflow and temporary deposition of fine-grained sediment from the impoundment \((6, 7)\). This would produce a local increase in the elevation of the downstream channel bed and floodplain, further enhancing flood potential. The sediment transport formula presented here is applicable to nearly the full range of lowland rivers. The impact of constructing or removing a dam in terms of sediment transport and subsequent stability of the channel may be evaluated with our new sediment transport relation and previous knowledge of the GSD of bed sediment.

**MATERIALS AND METHODS**

**Original form of EH formula**

The EH formula is a total load equation, comprising both the bed material part of suspended load and the bed load. Its dimensionless form reads \(C_t q_s^* = \alpha_{EH} \tau^{5/2} / \sqrt{g H D}\) is the dimensionless sediment discharge, that is, the Einstein number \(q_s = \frac{H}{gD}\) is the sediment discharge per unit width \((H\) is the water depth; \(U\) is the average flow velocity; and \(C\) is the volumetric sediment concentration, with contributions from both the bed material part of suspended load and the bed load); \(R\) is the sediment submerged specific gravity \(1.65\) for quartz; \(g\) is the gravitational acceleration; and \(D\) is the characteristic (nominal) grain size of bed material, and \(\tau^{5/2} / (\rho g RD)\) is the dimensionless form of total shear stress, that is, the Shields number, where \(\tau = \rho G U^2 = \rho u^2\) is the total shear stress \(G\) is the bed resistance coefficient and \(u^2\) is the shear velocity). For the normal flow condition (steady and uniform flow), \(\tau = \rho g HS\) where \(S\) is the channel bed slope, so \(\tau^{5/2} = HS/(RD)\) and \(C_t = gHS/U^2\).

While developing the original form of the formula, Engelund and Hansen \((33)\) used only four of the nine groups of data in the Guy-Simons-Richardson database \((41)\), obtaining a coefficient \(\alpha_{EH} = 0.05\) and power index \(n_{EH} = 2.5\). Here, all nine groups of data in the Guy-Simons-Richardson database \((41)\) are used, and the adjusted values for \(\sigma\) and \(n\) were determined to be 0.0355 and 3.0, respectively. The adjusted formula had only a slight difference from the original form in terms of predictive accuracy.

**Quasi-equilibrium sediment transport database**

Measurements of sediment load were collected at hydrological stations along the Huanghe since the 1950s. During sampling, several positions...
were located equally along a cross section, where three to five vertical sediment samples were collected. The vertical velocity profile, water discharge, water surface level, and surface gradient were also measured. The sediment concentration and GSDs of sediment samples (from both the water column and the bed) were subsequently analyzed; the grain size resolution was 5 μm. The sediment load data used here pertain to total load, which comprises the measured suspended load and a correction with the modified Einstein procedure (MEP) to account for the bed load. Because the measurement of both suspended sediment concentration and velocity extended down to a reference height not far above the bed, the MEP was used to calculate the bed load concentration and velocity below the reference height. On the basis of a theoretical model, the calculated bed load flux was added to the total sediment load. Additional details of this procedure were found in the study of Colby and Hembree (53). However, it should be pointed out that the component of bed load in the Huanghe was negligible. The average sediment concentration C and velocity U used here were the cross-sectional flux-averaged bed material load concentration and cross-sectionally averaged velocity, respectively.

The quasi-equilibrium database used here was selected from routine data collected during the 1980s, for which bed material GSDs are available, and before the construction of the Xiaolangdi Dam and reservoir. The criteria for data selection were that (i) water discharges and water surface level remain unchanged before and after the two-hour survey (steady flow) and (ii) the suspended load is consistent with other measurements at the same water discharge, surveyed at other times (19, 54). The quasi-equilibrium database covered a large range of flow and grain size conditions. Water depth ranged from 0.55 to 7.8 m, flow velocity ranged from 0.39 to 2.90 m/s, water surface slope ranged from 2 × 10−5 to 9 × 10−5, and the geometric mean grain size of bed material ranged from 15 to 158 μm. Therefore, the database well characterized the flow and bed material conditions during both nonflood and flood seasons.

**Bed bathymetry data**

The survey cruises of the lower Huanghe were conducted during the WSRS of 2015 and the nonflood (base flow) season of 2016. The survey site was at Kenli, near Dongying, China, close to Lijin Hydrological Station and 83 km upstream of the outlet. The bathymetric data were collected from a boat following a straight reach. The acquisition and processing procedures followed that of Nittrouer et al. (55). Because of the relatively shallow depth of the Huanghe compared to the Mississippi River, the multibeam survey swath width was considerably narrower, because this width is proportional to water depth beneath the multibeam; head; for the Huanghe, this depth was 1.0 to 4.0 m during the survey, whereas for the Mississippi River, it was 20 to 40 m.

**SUPPLEMENTARY MATERIALS**

Supplementary information for this article is available at http://advances.sciencemag.org/cgi/content/full/3/5/e1603114/DC1

**REFERENCES AND NOTES**


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