Supplementary Materials for

Large discrete jumps observed in the transition between Chern states in a ferromagnetic topological insulator

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DOI: 10.1126/sciadv.1600167

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section S1. Sample

Figure S1 is a photo of the device used in the experiment. The thin films of Cr-doped (Bi,Sb)$_2$Te$_3$ were grown in an MBE chamber to a thickness of 10 nm. After the growth was completed, the film was allowed to cool to room temperature and capped with about 3 nm of Al. The sample was then exposed to atmosphere to allow the Al capping layer to oxidize to form a protective layer of Al$_2$O$_3$. Using a sharp needle controlled by programmable stepper motors, we cut the film into the standard Hall bar shape shown in the figure. Indium contacts were pressed onto the film to break through the Al$_2$O$_3$ layer and achieve Ohmic contact with the 2D electron gas.

section S2. Jumps in negative field

The jumps appearing within the interval ($T_1$, $T_2$) when $H$ is increased in the positive direction are also observed when $H$ is decreased to negative values to induce the transition $C = 1 \rightarrow -1$. For completeness, we show the jumps in $R_{xy}$ and $R_{xx}$ in figs. S2 and S3, respectively. In fig. S2a, as $H$ is swept to more negative values (arrow) a large jump first appears at 65 mK. As discussed in the main text, small steps are visible at lower $T$ but they are distinct from the large jumps of interest here. In the window (70 mK, 145 mK), the large jumps dominate the transition. Again, the field $H_J$ at which the first jump is triggered moves systematically to more negative values as $T$ increases. As $T$ increases beyond 138 to 145 mK, the jump size rapidly decreases below our resolution.

In fig. S3a and b, the profiles of $R_{xx}$ vs. $H$ are similar to those shown in the main text. In the interval between 117 and 131 mK, successive jumps are in opposite direction, again similar to what is observed at positive fields.

The effects of raising $T$ are shown in fig. S4 for both $R_{xx}$ (panel a) and $R_{xy}$ (b), with $V_g$ fixed at -120 V. As noted in the main text, the coercive field $H_c(T)$, determined from the peak in $R_{xx}$, decreases significantly from $\sim 0.14$ T at 10 mK to $\sim 0.076$ T at 580 mK. The transition from the Chern state $C = -1$ to $C = 1$ also broadens considerably.

Surprisingly, the deviation in $R_{yx}$ from 1.0 is quite mild from 10 to 580 mK (panel a), whereas the background dissipation grows exponentially with $T$ (b).

fig. S1. Photo of the Hall bar used in the experiment. Black regions are the pressed indium contacts. The dark bar is 1 mm in length.
fig. S2. Curves of \( R_{xy} \) versus \( H \) for the transition between Chern states induced by sweeping \( H \) to negative values (arrow), with \( V_g \) fixed at \(-80 \) V. Panel a shows the curves for \( T \) at and below 70 mK. In the window 70–145 mK, large jumps are observed. The onset field of the first jump \( H_1 \) shifts to more negative values of \( H \) as \( T \) increases.

section S3. Tunneling out of metastable state

The tunneling of a magnetization configuration (as opposed to a single particle) recalls the false-vacuum problem first treated by Coleman and Callan (16, 17). The local magnetization is represented by a field \( \phi(\mathbf{x}, \tau) \) that “sees” a tilted double-well potential \( U(\phi) \), and is described by the action (16)

\[
S = \int dt \int d^3x \left[ \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 - U(\phi) \right] \quad (S1)
\]

(dissipation is absent and the spin wave velocity \( v_s \) is set to 1). The system is initially trapped in the higher well (the “false vacuum” Ref. 16). To calculate the escape probability, it is expedient to rotate into Euclidean space which inverts the potential well. Among the many paths linking the initial extremum to the final one in the inverted \( U \), minimization of the Euclidean action \( S_E \) gives the isotropic “bounce” solution (16, 17)

\[
\phi_B(\mathbf{x}, \tau) = -a \tanh \frac{\mu \rho}{2}, \quad \rho = \sqrt{|\mathbf{x}|^2 + \tau^2} \quad (S2)
\]

where \( \tau = it \), \( 2a \) is the well separation and \( \mu \) describes the well curvature. Equation S2 describes in Euclidean space (\( \mathbf{x}, \tau \)) an isotropic bubble of radius \( R \) (where the bounce occurs). Within the bubble, \( \phi \) is in the true ground state (with spin up). Rotating back to Minkowski space (\( \tau \rightarrow it \)), this solution describes tunneling “under the barrier” out of the metastable state within a seed bubble. After escaping, the spin-up bubble expands rapidly with velocity \( v_s \) until it consumes the whole sample in the absence of dissipation. The expansion, driven by the Zeeman energy released as the spins invert within the bubble, is relevant to our experiment.
fig. S3. Field profiles of $R_{xx}$ as $H$ is decreased through $-H_c$ at temperatures 10 to 83 mK (a) and 83 to 152 mK (b). Large jumps of either sign are observed in the window 117 to 131 mK.

section S4. One-wall model

We describe a simple model that captures qualitatively the observed jump pattern shown in Fig. 4 of the main text. We first consider the case in which the domain wall (DW) lies between the two pairs of Hall probes (fig.S5a). Assuming the absence of inelastic scattering, the resistances are determined by the overall scattering matrix at the domain wall (dashed rectangle in fig. S5a). The scattering matrix is given by

$$S = \begin{pmatrix} \sqrt{1-T} & -\sqrt{T} \\ \sqrt{T} & \sqrt{1-T} \end{pmatrix}$$  \hspace{1cm} (S3)

with $T$ the scattering probability across the DW. We neglect the scattering phases which are irrelevant here.

The full conductance matrix $G$, defined by $I = GV$ with $I = (I_1, I_2, I_3, I_4, I_5, I_6)^T$ and $V = (V_1, V_2, V_3, V_4, V_5, V_6)^T$, then reads

$$G = \frac{e^2}{h} \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & T & 1-T & 0 & -1 & 0 \\ 0 & 1-T & T & 0 & 0 & -1 \end{pmatrix}$$ \hspace{1cm} (S4)

The chemical potential at each contact determines the occupation of the edge channel flowing away from it. For e.g., in fig.5a, because backscattering is absent in the edge channel between contacts 1 and 2 (or, contacts 3 and 4), the occupation does not change and therefore $V_2 = V_1$ ($V_3 = V_4$). The change of occupation does occur after scattering at the DW and simple counting of the partition of current gives $V_2 = V_1 T + V_4 (1-T)$ and $V_6 = V_1 (1-T) + V_4 T$, which reproduces eq. (S5). We see that, if all the Hall probes are on one side of the DW, the longitudinal resistance $R_{14,23} = R_{14,65} = 0$, and the Hall resistance $R_{14,53} = R_{14,62} = \pm 1$ where the sign depends on the chirality of the edge states (expressing all resistances in units of $h/e^2$).
fig. S4. The effect of increasing $T$ on the curves of $R_{yx}$ (a) and $R_{xx}$ (b), with $V_g$ fixed at $-120$ V (upper edge of the optimal gating window). In Panel a, the plateau values of $R_{yx}$ remain close to $\hbar/e^2$ until $T$ rises above $100$ mK. A weak $n$-type slope implies that the excited carriers are $n$-type. Raising $T$ increases the background value of $R_{xx}$ exponentially. The coercive field $H_c(T)$, determined from the peaks decrease significantly with increasing $T$.

Solving the set of equations with the current conditions $I_1 = -I_4 = -I_0$ and the voltage probe conditions $I_2 = I_1 = I_3 = I_6 = 0$, together with a gauge choice $V_4 = 0$, we find

$$V_1 = V_2 = \frac{\hbar}{e^2} I_0, \quad V_3 = V_4 = 0, \quad V_5 = \frac{\hbar}{e^2} I_0, \quad V_6 = \frac{\hbar}{e^2} I_0 (1 - T)$$  \hspace{1cm} (S5)

The longitudinal and Hall resistances are then given by

$$R_{14,23} = \frac{V_2 - V_3}{I_0} = \frac{1}{T}$$  \hspace{1cm} (S6)

$$R_{14,65} = \frac{V_6 - V_5}{I_0} = \frac{1}{T} - 2$$  \hspace{1cm} (S7)

$$R_{14,53} = \frac{V_5 - V_3}{I_0} = 1$$  \hspace{1cm} (S8)

$$R_{14,62} = \frac{V_6 - V_2}{I_0} = -1$$  \hspace{1cm} (S9)

Hereafter, we take the value $T = 2/3$. After the first jump (panel a), we have $R_{14,23} = 3/2$, while $R_{14,65} = -1/2$. The Hall resistance $R_{14,53}$ jumps to 1 while $R_{14,62}$ remains unchanged at -1. The corresponding values for the DW placement shown in panel b are $R_{14,23} = R_{14,65} = 0$, while $R_{14,53} = R_{14,62} = 1$. These values are plotted in panels c and d.
fig. S5. Sketch of the device with a domain wall bisecting the film. The wall is assumed to lie between the Hall probes (6,2) and (5,3) after the first jump (a). After the second jump, the wall is between (2,6) and 1 (b). The red circles represent the contacts. The thick blue lines are the current-carrying edge channels. Current $I_0$ is injected at 1 and taken out at 4. With the assumed wall placements, c and d show the calculated resistances $R_{14,mn}$ taking $T=2/3$. The amplitudes are much larger than observed, but the signs of the changes to $R_{14,mn}$ for both jumps agree with experiment.

The model is too simple to be quantitative (it ignores the background dissipation and the formation of multiple domains and walls). The jump magnitudes are larger than observed by a factor of 2 (or more). However, the signs of the changes to the four resistances are faithfully reproduced for both jumps (Fig. 4).

section S5. Multiple domains and the network model

We first sketch the network model (34, 35) for treating the conventional integer QHE problem with disorder. The (scalar) disorder potential $V(x)$ creates a complicated undulating topography with energy contours that are predominantly closed loops. The centers of the cyclotron orbits follow classical trajectories that track the energy contours. Trajectories at an energy $E$ encircle local minima (maxima) of $V(x)$ if $E$ is far below (high above) the Landau band center $E_c$. These states are Anderson localized at very low $T$. At energy $E_c$, however, there exists extended states that traverse the sample.

A further complication is that tunneling between two distinct classical trajectories occurs if they get sufficiently close to each other. This alters the classical description in essential ways. Chalker and collaborators (34, 35) described a powerful network model that incorporates tunneling. The chiral, classical orbits are regarded as directed links carrying probability currents defined by a complex number $Z_i = |Z_i|e^{i\phi_i}$ ($i = 1, \ldots, 4$). In the tunneling regions (called nodes), incoming electrons on, say, links 2 or 4 can scatter into either of the outgoing links 1 and 3 with probabilities parametrized by an angle $\beta(E)$ that increases monotonically from 0 (at the band bottom) to $\pi/2$ (top).

The tunneling is maximal when $\beta = \beta_c \equiv \pi/4$. The transitions are described by the matrix equations (34, 35)

$$
\begin{pmatrix} Z_1 \\ Z_3 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} Z_4 \\ Z_2 \end{pmatrix} \quad (S10)
$$

$$
\begin{pmatrix} Z_2 \\ Z_4 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_3 \end{pmatrix} \quad (S11)
$$

Equation $S10$ applies to a node at which links 2 and 4 are incoming while $S11$ is for a node with links 1 and 3 incoming. The sample is comprised of a regular lattice of corner-sharing loops which are indexed by integers $x$ and $y$ (the nodes are the shared corners). Disorder is introduced by randomizing the phases $\phi_i$ in each link. At time step $L + 1$, the outgoing probability current in a particular link say $Z_i(x, y; L + 1)$ is a linear combination of the two incident links 2 and 4 (at time $L$). Using eq. $S10$, we express this relation as (35).
\[ Z_1(x, y; L + 1) = \sin \beta e^{i\phi_1} Z_3(x - 1, y + 1; L) + \cos \beta e^{i\phi_1} Z_4(x, y; L) \]  

(S12)

Propagation of current through the sample is described by the \(4 \times 4\) transfer matrix acting on the basis \((Z_1, Z_3, Z_2, Z_4)^T\) and given by (35)

\[
T = \begin{pmatrix} 0 & M \\ N & 0 \end{pmatrix}
\]

By applying Eq. S12 and similar equations for \(Z_2, Z_3, Z_4\), we find that the \(2 \times 2\) block matrices \(M\) and \(N\) are given by

\[
M = \begin{pmatrix} \sin \beta e^{i\phi_1} t_+^x t_+^y & \cos \beta e^{i\phi_3} \\ \cos \beta e^{i\phi_3} & -\sin \beta e^{i\phi_3} t_+^x t_+^y \end{pmatrix}, \quad N = \begin{pmatrix} \cos \beta e^{i\phi_2} & \sin \beta e^{i\phi_4} t_+^x t_+^y \\ \sin \beta e^{i\phi_4} t_+^x t_+^y & -\cos \beta e^{i\phi_4} \end{pmatrix}
\]

Here \(t_+^x\) are translation operators that change \(x\) by \(\pm 1\) via the action \(t_+^x Z_i(x, y) = Z_i(x \pm 1, y)\) (and likewise for \(t_+^y\)).

By sampling over large samples (up to \(128 \times 10^5\) cells), Chalker and Coddington (34, 35) demonstrated that the system displays extended states at the critical value \(\beta = \beta_c\), but is Anderson localized once \(\beta\) deviates from \(\beta_c\) by a few \%. A central result obtained is that the localization length diverges as \(\xi(\beta) \sim |\beta - \beta_c|^{-\nu}\), where \(\nu \sim 2.5\), very different from the classical value 1.

Next, we turn to the QAH problem with disorder. Wang, Lian and Zhang (WLZ) (Ref. 37) start with the Hamiltonian for the QAH system

\[
H_0(k) = v_F (k_x \sigma_1 - k_y \sigma_2) \tau_3 + \Delta \sigma_3 + m(k) \tau_2
\]  

(S13)

where \(v_F\) is the Fermi velocity and \(k = (k_x, k_y)\) the momentum of the surface Dirac states. \(\Delta\) describes the ferromagnetic exchange field. The coupling between the top and bottom surfaces of the thin film is represented by \(m(k) = m_0 + m_1|k|^2\). \(\sigma_i\) are Pauli matrices that act on the physical spins, whereas \(\tau_i\) are Pauli matrices that act on states in the top and bottom surfaces. Three types of disorder are present: a random vector potential \(A\) that couples to \(k\), fluctuations in the exchange term \(\Delta\) and a random scalar potential \(\delta V\). In the presence of these potentials, three types of domains defined by Chern numbers \(C = 0, \pm 1\) appear. Domains with \(C = 1\) are bounded by (say) clockwise chiral modes whereas domains with \(C = -1\) are bounded by counterclockwise chiral modes. The two domains are separated by regions with \(C = 0\). To describe tunneling between the chiral modes when they approach each other, WLZ map the QAH problem to the network model of Chalker et al., and argue that the same exponent \(\nu \sim 2.5\) also controls the divergence of the localization length \(\xi\) as the magnetization is varied across the coercive field.