ECONOMIC ANTHROPOLOGY

Settlement scaling and increasing returns in an ancient society

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A key property of modern cities is increasing returns to scale—the finding that many socioeconomic outputs increase more rapidly than their population size. Recent theoretical work proposes that this phenomenon is the result of general network effects typical of human social networks embedded in space and, thus, is not necessarily limited to modern settlements. We examine the extent to which increasing returns are apparent in archaeological settlement data from the pre-Hispanic Basin of Mexico. We review previous work on the quantitative relationship between population size and average settled area in this society and then present a general analysis of their patterns of monument construction and house sizes. Estimated scaling parameter values and residual statistics support the hypothesis that increasing returns to scale characterized various forms of socioeconomic production available in the archaeological record and are found to be consistent with key expectations from settlement scaling theory. As a consequence, these results provide evidence that the essential processes that lead to increasing returns in contemporary cities may have characterized human settlements throughout history, and demonstrate that increasing returns do not require modern forms of political or economic organization.

INTRODUCTION

Many studies over the last few decades have demonstrated that average properties of contemporary urban settlements—from socioeconomic outputs to land area to the extent of infrastructure—vary systematically and predictably with population size (1–6). For example, measures of the physical extent of urban infrastructure increase more slowly than city population size, thus exhibiting economies of scale. On the other hand, various socioeconomic outputs increase faster than population size and thus exhibit *increasing returns to scale*. Recent theory, building from comparative analyses of large data sets for many urban systems around the world, has proposed that these properties of modern cities take a simple mathematical form and emerge from a few general principles of human social organization (2). This view posits the primary role of cities in human societies as social reactors: Larger cities are environments where a larger number of social interactions per unit time can be supported and sustained. This generic dynamics, in turn, is the basis for expanding economic and political organization, such as the division and coordination of labor, the specialization of knowledge, and the development of (hierarchical) political and civic institutions. Thus, whereas the expression of these activities is local and reflects history and culture, the larger cities in any urban system, on average, share common characteristics as they magnify social interaction opportunities (2, 3, 7) and provide better matching complementarities (8), thereby increasing the productivity and scope of material resources and human labor (2, 7, 8).

An important aspect of these ideas is that the theoretical derivation of scaling relations does not invoke specific characteristics of modern economies, industrialization, or global trade, but instead relies only on basic self-consistent characteristics of human social networks embedded in space. Consequently, these models are potentially applicable to ancient (and even non-urban) settlement systems and make a set of integrated and novel predictions for the structure and function of these systems that can be tested using archaeological evidence. We have previously introduced settlement scaling theory and examined the extent to which spatial economies of scale characterized the pre-Hispanic Basin of Mexico (BOM) (7). Here, we review this previous work and discuss the relationship of settlement scaling theory to existing models of spatial economics. Then, we investigate two ways in which increasing returns to scale are expressed by archaeological data from the pre-Hispanic BOM (Fig. 1) by analyzing the scaling properties and statistics of monument construction and house sizes versus settlement population. We find not only that increasing returns were present in the pre-Hispanic BOM but also that the measure of such returns (elasticity) has the same numerical value predicted by settlement scaling theory and observed in modern urban systems. On the basis of these combined results, we propose that scaling phenomena in all human societies emerge from the same essential processes, and that settlement scaling theory provides a unifying view of these patterns and a novel theoretical framework for the interpretation of archaeological data.

Previous research

We begin by briefly reviewing settlement scaling theory and its relation to standard models of land use in cities and to extensive empirical observations of modern urban areas. This allows us to derive in simple terms the main expectations of scaling theory for settlements in the archaeological record as a function of their population size. We then introduce the main characteristics of the BOM settlement surveys emphasizing how they enable the creation of independent measures of population and occupied land area, leading to a test of expected spatial economies of scale following from scaling theory. These arguments set up the necessary formal expectations for data expressing potential increasing returns to scale, characterizing social and economic production in ancient societies.

Settlement scaling theory

Simple spatial models of settlements have a long history in geography and economics, starting with the von Thünen model (for the isolated

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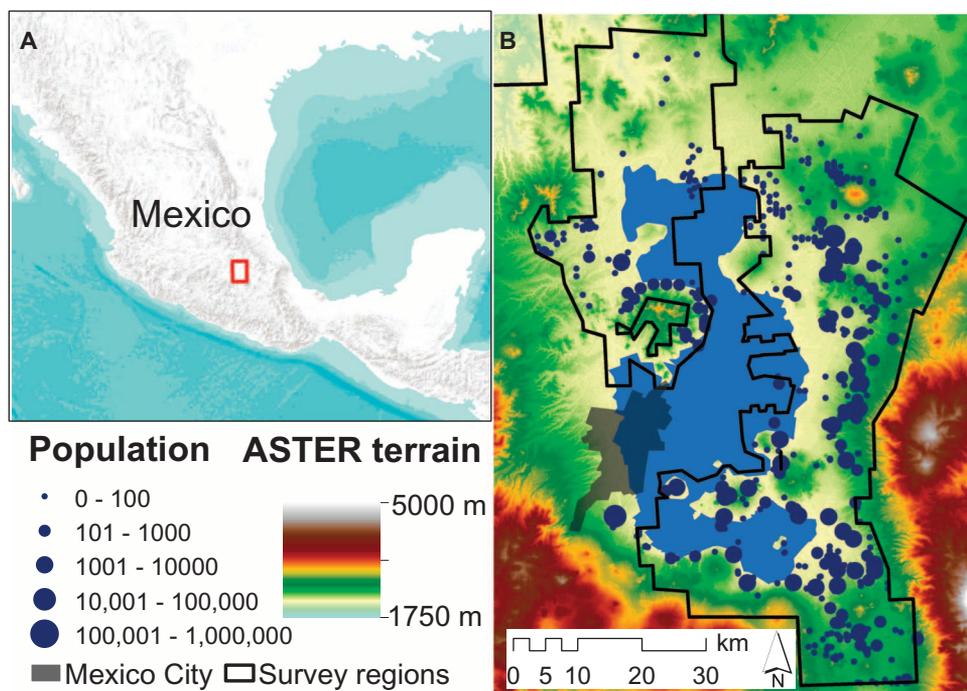*Corresponding author. E-mail: scott.ortman@colorado.edu

Fig. 1. The BOM. (**A**) Location within Mexico. (**B**) Settlements dating to the Formative period (outline shows surveyed area; circle size is proportional to population; colors denote elevation; gray area shows the extent of Mexico City in 1964). Today, settlement covers the entire basin, and the lake has been drained. See the Supplementary Materials for image sources.

form of spatial equilibrium where, on average, the benefits of social interaction balance movement costs for a given population size N. In this case, the cost of movement c is set by the energetic cost of walking ε (measured in cal/length) times the diameter of the circumscribing area L, which is proportional to the only characteristic length scale in the problem: the square root of the area, $c = c_0 \varepsilon A^{1/2}$. This is true in many different geometries and does not require that the settlement be circular or the transportation radial (9, 10). The dimensionless constant c_0 is a number of order unity, as demonstrated by studies exploring movement in different urban forms (10). The average social benefits of interaction with others y are then estimated through the average productivity of each interaction g times the ratio of urban volume covered by a person in the settlement over its area, times the number of people in the settlement. If we parameterize these quantities by the distance at which interaction occurs a_0 (a cross-section in the language of physics) and the distance travelled over the given period l, we obtain $y = (ga_0l/A)N$. Equilibrium between social net benefits and movement costs, $c(N) = y(N)$, then leads to $A(N) = aN^\alpha$, where $a = (ga_0l)/(c_0\varepsilon)^\alpha$ and $\alpha = 2/3$.

This simple picture needs to be elaborated as settlement densities increase and urban space becomes more structured and differentiated (2). The main feature of such changes is the explicit appearance of spaces dedicated to flows, such as streets and waterways. As this happens, dwellings align along these transportation networks, as can be seen in maps of many ancient and modern cities. This change toward a *networked* organization with size and density has been noted in archaeology by Flannery (11), who suggested that it may be a general feature of human settlement growth.

Such spatial organization has a different geometry from the amorphous settlement and leads to different scaling exponents. To extend the amorphous settlement model to larger and denser "networked" settlements, we assume that infrastructural space d is set aside on a per capita basis, proportional to the overall density of settlement, such that $d = \rho^{-1/2}$, where $\rho = A/N$ [more complex and thorough models can also be developed that justify these simple derivations (2)]. Thus, the total area of the infrastructural network A_n is proportional to the population times the square root of area over population, $A_n \sim A^{1/2} N^{1/2}$. Substituting $aN^{2/3}$ for A leads to $A_n \sim a^{1/2} N^{5/6}$, the relation observed for infrastructural quantities in contemporary metropolitan areas (2, 6).

Finally, we assume that total socioeconomic outputs, whether positive in the form of economic production or innovation, or negative in the form of contagious diseases or interpersonal violence [see (5)], are proportional to the total number of interactions that take place in a settlement, with technology and culture influencing only the productivity (and cost) of each interaction. Given this, we can derive the

state). More recent models apply similar ideas to the structure of cities, due to Alonso and others (9). These models are most frequently invoked to derive patterns of land use as the result of the interplay between land rents and transportation costs in monocentric settlement geometries (9).

Settlement scaling theory retains some of the characteristics of these models but generalizes them in a few ways that apply to more realistic situations. First, settlement scaling theory considers situations where circular symmetry is not necessary (there is no single central market) by requiring only that a number of social encounters are realized over a specific geometry per unit time. Crucially for archaeology, settlement scaling theory also does not specify the type or value of goods to be transported: The problem of what can be produced and spent is instead determined more abstractly as the result of socioeconomic rates of interaction. What is specified in our approach is the universal dependence of the rates at which goods are produced (and consumed) on population size (elasticities) (2, 7). As such, the ideas that underlie settlement scaling theory are more general than traditional models of land use economics and can be developed and tested in the context of ancient societies and archaeological data.

In (2, 7), we propose that human settlements are, first and foremost, social networks embedded in space. In this view, settlements are containers within which a population interacts frequently, with internal social interactions far outnumbering external ones. From this very simple general picture, one can derive an expected scaling relation between total settlement land area and population. To see this, consider a settlement with relatively low density of occupation that is not especially organized or planned spatially. We call such settlements *amorphous* and posit that their spatial extent is set by a simple

expected socioeconomic output of a settlement Y, relative to others, by multiplying the per capita benefit of interaction by the population, $Y = yN = ga_0lN^2/A$, and then substituting the relation for the infrastructural area A_n for A. This simplifies to $Y = GN^2/A_n \sim N^2/N^{5/6} \sim N^{7/6}$, where $G = ga_0l$. This expectation is consistent with observations across many contemporary urban systems (2), and its test in the context of archaeology is the focus of the new work presented in this paper.

An important feature of these models is that their assumptions and input parameters are very general and are not specific to modern political or economic organization. Exponents are "universal" because they are set by the congruence of the geometries of spatial and social networks, under the assumption of an equilibrium between centripetal social interactions and centrifugal costs (2, 9), as well as the requirement that such arrangements stay open-ended relative to settlement size (2). Simple elaborations of these arguments further derive expected patterns of professional diversity (division of labor) (12), labor productivity (13), and urban area production (14) that are all consistent with empirical evidence from modern cities.

Population and settled area

The population-area scaling relations discussed above apply to settlements for which it is reasonable to model the settled area as the container within which the resident population interacts on a regular basis. It proposes that such settlements tend to grow in ways that balance the costs of moving within the settlement with the benefits of the resulting social interactions. Thus, settlements whose spatial arrangements balance these costs and benefits should exhibit a specific and consistent overall average relationship between the resident population and settled area. Specifically, settlement scaling theory predicts that the exponent relating population to settled area for "interaction container" settlements should fall in the range between 2/3 and 5/6, with this exponent being closer to 2/3 among small, amorphous settlements and closer to 5/6 among larger, networked settlements. It also suggests the area taken up by an individual in the smallest such settlements derives primarily from travel costs (walking, in this context) and the average (energetic) benefit of social interactions. Technologies that reduce transportation costs or increase the effectiveness of interaction should increase this baseline area, but factors that influence the rate of energy capture by primary producers should not. This is because the movement and exchange of agricultural produce provide a stronger constraint on energy flows in social networks than agricultural production itself. Thus, our models predict that the prefactor of the scaling relation between population and settled area should be respon-

sive to changes in within-settlement transport technology, but not to changes in agricultural productivity.

In (7), we tested these expectations by comparing the populations and settled areas of BOM settlements dating from four pre-Hispanic cultural periods. We now briefly describe these periods and some of the key characteristics of the surveys. The Formative period (1150 BCE to 150 CE) saw the beginnings of detectable settlements and the rise of local polities; the Classic period (150 to 650 CE), the political and economic dominance of Teotihuacan ($N \approx 100{,}000$); the Toltec period (650 to 1200 CE), the formation of a number of small competitive polities; and the Aztec period (1200 to 1520 CE), the unification of these into an empire centered on Tenochtitlán ($N \approx 200{,}000$) that was in place at the time of the Spanish conquest. We also compared two size classes corresponding to "amorphous" ($N < 5000$) and networked ($N \geq 5000$) settlements and 1960 census data from the same area. For details of our data selection and grouping criteria, see (7). The key characteristic of the BOM archaeological surveys that allowed us to assess the population-area relationship was that it estimated population densities through potsherd densities according to a graded scale. In (7), we performed several statistical tests of the independence of this measure from total area, and of the correlation of the resulting population estimates with more direct dwelling counts, where available (see also Materials and Methods).

Portions of our previous results, updated to take a few minor data edits into account, are presented in Table 1. These results show that the exponent of the average scaling relation lies within the interval $2/3 \leq \alpha \leq 5/6$ for each of the four major cultural periods. They also illustrate the transition from 2/3 to 5/6 for amorphous versus networked settlements predicted by our theory. Finally, they show that there is no clear evidence for change in the prefactor of the average scaling relation across pre-Hispanic periods, despite significant changes in maximum agricultural yields. However, we also find that the prefactor for the 1960 data is significantly larger than those observed across pre-Hispanic periods, consistent with the major innovations in transportation technology, and other contextual changes, of more recent times.

RESULTS

We now show that, in addition to spatial economies of scale, increasing returns to scale are also apparent in the BOM survey data. First, we compare the populations and time periods of political units with

Table 1. Population-area scaling analysis results. The yields (kg/maize per hectare) of the most productive agricultural strategies for the pre-Hispanic periods are as follows: Formative, 700; Classic and Toltec, 1400; and Aztec, 3000 (28). CI, confidence interval.

Group	Sites	a (ha)	95% CI	α	95% CI	r^2
Formative (1150 BCE to 150 CE)	230	0.195	0.160–0.238	0.711	0.673–0.749	0.855
Classic (150–650 CE)	272	0.221	0.174–0.279	0.632	0.583–0.681	0.707
Toltec (650–1200 CE)	484	0.210	0.180–0.244	0.718	0.684–0.753	0.777
Aztec (1200–1520 CE)	546	0.177	0.156–0.201	0.764	0.734–0.793	0.830
1960 Census	181	0.445	0.250–0.945	0.641	0.552–0.729	0.532
Amorphous ($N < 5000$)	1510	0.237	0.217–0.259	0.671	0.651–0.691	0.741
Networked ($N \geq 5000$)	22	0.109	0.009–1.303	0.853	0.598–1.109	0.709

Table 2. Estimated scaling parameters for socioeconomic outputs with population. For the first analysis, the independent variable is the population of the political unit; for all others, the independent variable is the settlement population.

Dependent variable	Sample	Prefactor	95% CI	Exponent	95% CI	r^2
Civic mound volume/year	48	$Y_0 = 0.0021$	0.0006–0.0070	$1 + \delta = 1.177$	1.028–1.327	0.852
Domestic-mound area (m^2)	80	$y_0 = 168.6$	93.3–304.8	$\delta = 0.190$	0.083–0.298	0.863
$Y = m * N$	80	$Y_0 = 168.6$	93.3–304.8	$1 + \delta = 1.190$	1.083–1.298	0.863
$G = m * A/N$	80	$G_0 = 28.71$	12.88–63.97	$\gamma = 0.037$	−0.108 to 0.182	0.003

public monument volumes to illustrate that larger political units produced such structures at faster rates per capita. Second, we compare settlement populations with domestic mound areas and find that houses in more populous settlements were larger on average. In both cases, we observe scaling exponent values that are statistically indistinguishable from those observed in modern urban systems and that are consistent with quantitative predictions of settlement scaling theory (*2*, *7*).

Polity population and monument construction

Pre-Hispanic Mesoamerican societies are well known for their monumental architecture, especially pyramids, administrative palaces, and plaza-focused buildings. Several factors allow us to treat the total volume of public monuments in administrative and ceremonial centers as a measure of the total production of corvée labor pools drawn from the subject population over a period of time (see the Supplementary Materials for details). Given these linking arguments, settlement scaling theory predicts that average public monument construction rates should be proportional to the population of subject settlements to the 7/6 power, or more formally $Y = G \, N^2/A_n \sim N^2/N^{5/6} \sim N^{7/6}$.

In the Supplementary Materials, we provide a detailed account of our procedures for estimating the volumes of civic-ceremonial structures and the size of the subject populations that contributed labor to these monuments during different cultural periods. Essentially, we combine archaeological studies of political organization and correlations between BOM survey data and ethnohistoric sources to create groups of settlements, populations, and monuments for specific political units and archaeological periods (*15*–*21*). The resulting data set is presented in table S2. These data are insufficient for a detailed time-series analysis, but the average scaling relation between political unit population and public monument construction rates (total volume/ years in period) across all periods (Table 2 and Fig. 2A) indicates that the exponent of this relation $1 + \delta \sim 7/6$, as predicted by theory. These results suggest that larger corvée labor groups generally produced more per person and per unit time than smaller groups, with the relative degree of benefit numerically identical to that observed for socioeconomic rates in modern urban systems (*2*, *3*).

It is also apparent in Fig. 2A that political units from different periods follow essentially the same scaling relation, characterized by a single prefactor representing the baseline productivity of an individual [that is, $Y(N \rightarrow 1)$]. In the Supplementary Materials, we show that there is no evidence for change in scaling parameters through time. Specifically, we show in table S5A that the null hypothesis of no change in scaling parameters through time is far more likely than the alternative. This in turn suggests that there was little change in the technology and energetics of public monument construction over time. As a result, the remarkable concentrations of public monuments
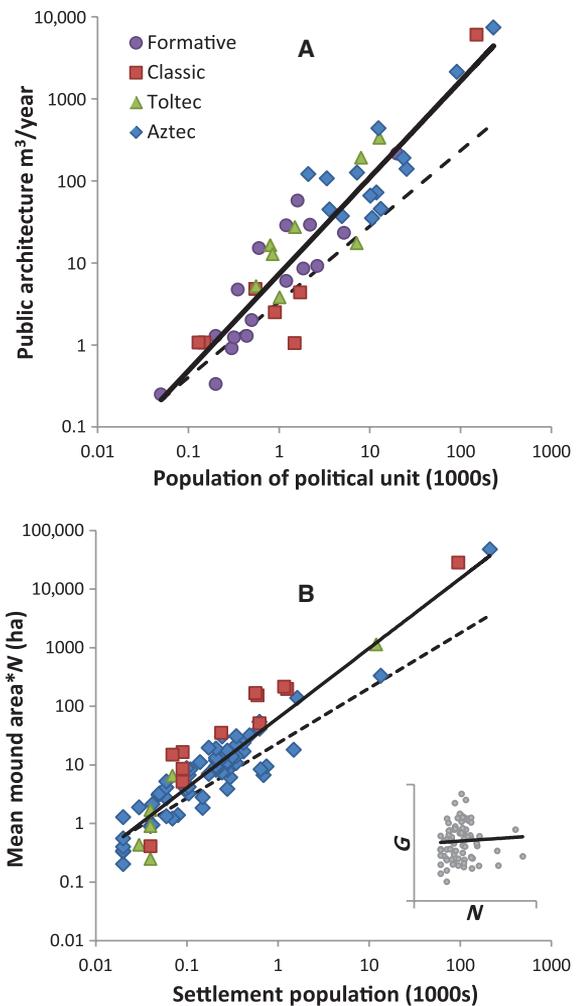

Fig. 2. Super-linear scaling of socioeconomic rates with population. (**A**) Political unit population versus public monument construction rates. (**B**) Settlement population versus total domestic mound area. Symbols denote time periods, solid lines show power law fits from ordinary least-squares (OLS) regression of the log-transformed data, and dashed lines represent proportionate (linear) scaling. Inset shows the independence of average G on N, where $G = A/N *$ (mean domestic mound area) (see also Table 2).

in political capitals can be attributed to the populations these authorities regulated and the returns to scale associated with the resulting corvée labor forces.

In analyzing the data this way, we do not mean to suggest that there was no variation in the annual labor tax, the proportion of

population that performed corvée labor, the length of time over which monuments were built, or the technology of monument design and construction. Undoubtedly, there was variation in all these factors. Yet, it is important to remember that these data vary over five orders of magnitude and as a result the overall scaling relationship is fairly robust to variation in these factors, the impact of which is summarized by the residuals of individual cases to the average scaling relation in Fig. 2A (5). That the results are so well behaved despite the wide chronological and demographic scope of the data suggests that the dominant factor behind monument construction rates was increasing returns on the size of corvée labor pools.

Settlement population and house area

Settlement scaling theory predicts that average per capita productivity should increase with settlement population to the power $\delta \sim 1/6$, because the productivity of an individual in a social network is proportional to the number of interactions that person has with others. Stated more formally, $y = G\,N/A_n \sim N/N^{5/6} \sim N^{1/6}$, where N/A_n reflects the density of people with respect to the infrastructural area through which people move and $G = ga_0l$, with a_0l reflecting the area covered by an individual's daily movements and g reflecting the average productivity of daily interactions (2, 7). Thus, the average productivity of an individual $y = y_0 N^\delta$, where $\delta = 1/6$. In turn, this implies that the total average productivity of a settlement is given by $Y = yN = y_0 N^{1+\delta}$. Per household, production should scale the same way as per capita production because in most societies, households are the basal units of production and consumption (see the Supplementary Materials for more details).

We treat the surface area of a domestic residence as a proxy for the productivity of the associated household unit (see the Supplementary Materials for background and justification). Given this association, settlement scaling theory leads to four expectations. First, following the typical, heavy-tailed distribution of wealth and income in contemporary societies [see, for example, (22)], the overall distribution of house areas should be approximately log-normal. Second, mean house area m should scale with settlement population N raised to the 1/6 power, and thus, the product of mean house area and settlement population mN should scale with settlement population to the 7/6 power. Third, the product of mean house area and settled area per person, mA/N, should vary independently of settlement population. This is because these two measures represent the productivity of daily interaction and the area over which these interactions occur, respectively, and thus, their product is an estimate of G, which should be independent of settlement size on average. Finally, the residuals of the average scaling relation between settlement population and mean house area should be approximately log-normally distributed. This is a consistency check necessary to confirm that there is no heteroscedasticity in the data (scale dependence of the log-variance). Because the exponent of house sizes is small ($\delta \sim 1/6$), it is especially important to confirm that the residuals from the best-fit line are approximately normal, and thus, the fluctuations in mean house area across

settlements are log-normal, as is typically found in analyses of contemporary data (23).

The primary data source for this analysis consists of surface areas of domestic mounds (including elite residences and palaces) recorded in BOM survey reports, with additional data added for specific sites from the literature (see the Supplementary Materials for details). Using these data, we computed m for interaction container settlements that are at least 1 hectare in area, possess well-preserved architectural remains, and are associated with at least two measured domestic mounds. Then, we estimated the total production of that settlement as $Y = mN$, and G as $G = mA/N$. The resulting data set is presented in detail in table S3.

Once again, there are insufficient data for time-series analysis, but patterns in the pooled data are consistent with expectations. First, the distribution of all domestic mound areas in the analysis (Fig. 3A) is approximately log-normal, with most deviations from log-normality deriving from expedient rounding of mound dimensions by fieldworkers (for example, 5 m × 5 m = 25 m^2, 20 m × 20 m = 400 m^2, 30 m × 30 m = 900 m^2, and so forth). Second, the average scaling relation between N and m and N versus $Y = mN$ (Table 2 and Fig. 2B) shows that mean household productivity scales with settlement population as expected. Third, our measure of G is independent of settlement population (Table 2 and Fig. 2B, inset). These results indicate that per-household productivity was higher in larger settlements, with the degree of increasing returns offset by the economy of scale in population density. Finally, the distribution of standardized residuals from the average scaling relation between m and N is approximately normal (Fig. 3B), given deviations derived from expedient rounding by fieldworkers. Thus, the fluctuations from the average scaling relation are approximately log-normally distributed, despite the partial and imprecise nature of the data, the slow rate of increase in m with N, and modest overall correlation between these variables.

These results suggest that the average productivity of households in the BOM generally increased with settlement size in the manner predicted by settlement scaling theory. However, as was the case for public monument construction rates, the data from all periods appear to follow the same scaling relation and are reasonably well characterized by a single prefactor that applies across periods. In the Supplementary Materials, and especially table S5B, we show that once again there is no evidence of change in the scaling parameters for this relationship
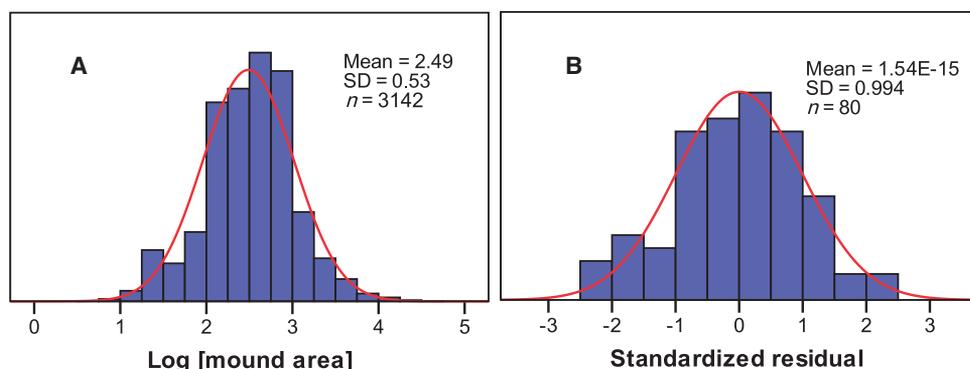
Fig. 3. Histograms of domestic-mound areas. (**A**) Distribution of log-transformed domestic mound areas across all sites. (**B**) Distribution of residuals from OLS regression of log [settlement population] versus log [mean house area]. Note that both distributions are approximately normal [Kolmogorov-Smirnov (Lilliefors) test results are $P < 0.001$ for (A) and $P = 0.2$ for (B)].

through time. This suggests that the baseline productivity of individuals $Y(N = 1) = y_0$ did not change appreciably over time. Yet, these results also suggest that the average wealth of households could have varied as a result of changes in the distribution of settlement sizes during different periods. In this way, the material conditions of life would have improved for many households, even if there were no significant improvements in baseline productivity. We consider this issue in more detail below.

DISCUSSION

We have outlined an approach to the analysis of ancient societies rooted in complex systems concepts and applied this framework to data from the pre-Hispanic BOM to investigate the degree to which patterns of contemporary urban scaling also characterized this ancient society. We are not the first to notice the fundamental importance of settlement population in structuring many observable properties of human societies [see, for example, (24–27)]. What we have attempted to do is extend this work by proposing that the exponents that describe the average relative effects of settlement population for various aggregate socioeconomic measures take specific numerical values, and these values can be derived from models that frame human settlements as social reactors embedded in space. In other words, we do not merely suggest that there are power-law relationships between population and a variety of other measures across settlements in a system, but we predict that the exponents of these relationships should have specific values. Further, we suggest that the prefactors of average scaling relations are given by specific combinations of additional parameters and, thus, that it is possible to infer the values of combinations of these parameters from empirical estimates of these prefactors. This provides a general testable framework in the context of archaeology characterized by a set of key predictions for how settled space, population, infrastructure, and rates of socioeconomic production are all interdependent. In the present article, we were able to show not only how archaeological observables characterizing social productivity express increasing returns to scale in the settlements of the BOM but also how such relations relate, as predicted by theory, to patterns of spatial settlement and their statistics.

The analysis developed here requires a number of assumptions, linking arguments and data selection criteria that should be investigated further. It is also important to recognize that, even if the scaling phenomena reported here prove to be universal, scaling relationships may not be universally observable due to the nature of archaeological and historical data. Nevertheless, settlement scaling theory should apply to any society due to its basic underlying assumptions, and the BOM surveys represent one of the most systematic documentations of an ancient non-Western civilization ever accomplished. Thus, the fact that the archaeological record of this society can be shown to exhibit the same quantitative relationships between population, infrastructure, and socioeconomic outputs predicted by theory and observed in modern urban systems is, in our view, a striking and exciting result.

Settlement scaling theory proposes that one can use a variety of aggregate quantities as measures of infrastructure, socioeconomic outputs, social network connectivity, and their evolution over time in any society. In this case, we have shown that in the pre-Hispanic BOM, larger population aggregates used space more efficiently, produced public goods more rapidly, and were more productive per household. Further, the congruence of these results with theory suggests that the benefits of scale across all these domains ultimately derive from the properties of strongly interacting social networks embedded in structured spaces. This reinforces our view that human settlements of all times and places function in the same way by manifesting strongly interacting social networks, thus magnifying rates of social interaction and increasing the productivity and scope of material resources, human labor, and knowledge (2, 7, 13).

In addition to the constancy of scaling exponents, our analyses suggest a, perhaps surprising, consistency in scaling prefactors across cultural periods. Scaling theory predicts that scaling prefactors should change in cases where transport costs or the average productivity of individual interactions changes over time. (In contemporary societies, rates of change are very fast, of the order of a few percent a year.) Such changes could arise from technological innovations such as beasts of burden or wheeled vehicles, changes in information technology such as currency or literacy, or any number of factors that facilitate the flow of goods and services through a social network. The system-level effects of such innovations should be perceptible through scaling analyses of quantities measured before and after their appearance, as our comparison of the pre-Hispanic and 1960 census data illustrates. Given this, the fact that such changes are not evident across the pre-Hispanic periods suggests that one of the two sources of modern economic growth—increases in baseline productivity—was limited in this society (14). Agricultural production per unit of land clearly improved over time (16, 28), and this had a significant impact on the maximal size of settlements and their spatial distribution, but the overall productivity of an individual working alone appears to have been relatively constant. As a result, any changes in per capita economic output over time are most likely traceable to changes in the size and density of social networks in larger settlements. This may have affected average household "income" during periods when sociocultural and political institutions enabled larger fractions of the population to live in larger settlements, but changes in the material conditions of life would have depended on the economies and returns associated with social interaction as opposed to increases in essential labor productivity. This is potentially a striking and important realization with wide-ranging implications for our understanding of human civilization.

If, in fact, larger social networks are intrinsically more efficient and productive, there is at least the potential for the majority of individuals in a society to benefit from increases in their scale and scope. Whether individuals actually do depends on the way in which the system-level benefits of scale are distributed among the individuals comprising these networks. Our analyses suggest that the material conditions of life did improve for that fraction of the BOM population that lived in larger settlements, but the baseline area per person, and roofed space per person, did not change much over time in small settlements devoted primarily to food production. This suggests that there was little change in the conditions of life for primary producers in this society, regardless of the way the benefits of scale were distributed among the more urbanized population. During periods of larger-scale social coordination, the BOM population was more productive and more efficient overall, but disparities in production (income) were high in all periods, with 40 to 50% of total house area encompassed by the top decile of households. This is comparable to levels of income disparity seen in the contemporary United States [see table S4 and (29)].

The conclusion we draw from these findings is that the (latent) benefits of scale for human groups are universal and are likely responsible for the long-term increase in human group sizes worldwide (30), but there are fewer constraints on how these benefits are distributed among individuals in groups. Thus, we suspect that one of the most fundamental dynamics in early civilizations was the age-old tension between the benefits of scale and the allocation of these benefits. In this particular case, it appears that reinvestment of surpluses toward innovations that increased baseline productivity was quite rare, and that this choice limited intensive economic growth in the long run. It is also important to emphasize that we have focused on increasing returns to scale in economic production, but negative effects of intensified social interaction, such as contagious disease and violence, also increase super-linearly with population in contemporary urban systems [see (5)] and can reduce the net benefit of social interactions encapsulated in the parameter g, or indeed destroy the necessary conditions for large-scale settled sociality (2). One might expect the same to have occurred in ancient societies, and future research could investigate this possibility.

The findings and suggestions for future research discussed in this paper reinforce our view that the archaeological record presents a vast archive of information on the determinants of socioeconomic development and that settlement scaling theory provides a useful framework for organizing various findings and examining potentially universal patterns in these processes while also allowing some degree of historical contingency. We thus believe that settlement scaling theory offers a means through which archaeology can make a broader contribution to the social sciences and perhaps even to contemporary policy (30–32).

MATERIALS AND METHODS

Data sources
We use settlement data from archaeological surface surveys conducted in the BOM, the epicenter of pre-Hispanic Mesoamerican civilization. These surveys took place between 1960 and 1975, before the destruction of many sites by the expansion of modern Mexico City. Figure 1 shows the location of our study area, the surveyed areas relative to Mexico City when the surveys took place, and the distribution of pre-Hispanic settlements for the Formative period. We compiled a database of information for some 4000 archaeological sites resulting from these surveys, beginning with existing digital compilations (33, 34) and adding information from the original survey reports (15, 35–42). We also added data for a few important sites, some of which were outside the survey area, based on information in the literature (43–48). In addition, we tabulated the dimensions of civic-ceremonial mounds reported in the survey volumes or in other sources (16, 44, 45, 49, 50), and we associated Aztec-period settlements with native political units based on ethnohistoric information summarized by Hodge and others (17–19). The resulting database contains information on the settled area, population, time period, location, functional classification, political affiliations, and architectural remains of every recorded settlement. For additional details and background on this database, see (7) and the Supplementary Materials.

Site population and parameter estimation
A potential problem with the BOM survey data is that the method used to estimate population for most sites was not independent of the settled area (16, 35). This method involved (i) determining the extent of the surface artifact scatter for each period of occupation by mapping its boundary on low-altitude aerial photos, (ii) assigning each scatter to one of a series of artifact density classes based on the observed potsherd density within the scatter, and (iii) multiplying the extent of the scatter for each period by a population density derived from associations of surface potsherd densities with population densities of various settlement types in 16th and 20th century records from the area. This method ensures that there will be a relationship between the settled area and settlement population. In (7), we show that the estimates produced by this area-density method are nearly identical to those produced using house-counting methods that rely on the count or surface area of residential mounds at well-preserved sites in lieu of settled area. Thus, there is a basis for viewing the BOM population estimates as reasonably accurate in both a relative and an absolute sense. In all analyses, we use OLS regression of the log-transformed data to estimate scaling exponents, prefactors, and confidence intervals for these parameters.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/1/1/e1400066/DC1
Materials and Methods
Fig. S1. The Basin of Mexico.
Table S1. Civic-ceremonial structures at Tenochtitlan-Tlatelolco.
Table S2. Civic-ceremonial architecture volumes and associated subject populations.
Table S3. Mean domestic-mound areas and settlement populations.
Table S4. Domestic mound area distributions in the Basin of Mexico through time.
Table S5. Within-period scaling analyses.

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