Verifying Heisenberg’s error-disturbance relation using a single trapped ion

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Heisenberg’s uncertainty relations have played an essential role in quantum physics since its very beginning. The uncertainty relations in the modern quantum formalism have become a fundamental limitation on the joint measurements of general quantum mechanical observables, going much beyond the original discussion of the trade-off between knowing a particle’s position and momentum. Recently, the uncertainty relations have generated a considerable amount of lively debate as a result of the new inequalities proposed as extensions of the original uncertainty relations. We report an experimental test of one of the new Heisenberg’s uncertainty relations using a single \(^{40}\)Ca\(^{+}\) ion trapped in a harmonic potential. By performing unitary operations under carrier transitions, we verify the uncertainty relation proposed by Busch, Lahti, and Werner (BLW) based on a general error–trade-off relation for joint measurements on two compatible observables. The positive operator-valued measure, required by the compatible observables, is constructed by single-qubit operations, and the lower bound of the uncertainty, as observed, is satisfied in a state-independent manner. Our results provide the first evidence confirming the BLW-formulated uncertainty at a single-spin level and will stimulate broad interests in various fields associated with quantum mechanics.

INTRODUCTION

Heisenberg’s uncertainty relation (1) is one of the cornerstones in understanding quantum mechanics. In most textbooks, the uncertainty relation is quantified by the standard deviations (SDs) of the measured variables, such as \(\Delta P \Delta Q \geq \hbar / 2\) (where \(\hbar\) is the Planck constant divided by \(2\pi\)), with \(\Delta P\) and \(\Delta Q\) being the SDs of two noncommuting operators \(\hat{P}\) and \(\hat{Q}\). This definition, which implies that the measurements of \(\hat{P}\) and \(\hat{Q}\) are performed on an ensemble of identical prepared quantum systems, describes a preparation uncertainty (2–4), which is actually different from the original spirit of Heisenberg’s idea. A correct understanding of Heisenberg’s uncertainty relation should be based on the observer’s effect; that is, the accuracy of an approximate position measurement is related to the disturbance of the particle’s momentum (1). This is a measurement uncertainty, also called the error-disturbance relation (EDR). For the above defined variables \(\hat{P}\) and \(\hat{Q}\), which are not restricted to describing the position and momentum of a particle, Heisenberg’s EDR, as strictly proven recently (5), is quantified as \(\epsilon(\hat{P}) \xi(\hat{Q}) \geq \hbar / 2\), where \(\epsilon(\hat{P})\) is the measurement error of the observable \(\hat{P}\), and \(\xi(\hat{Q})\) is the disturbance magnitude of \(\hat{Q}\) induced by the measurement.

Both the preparation uncertainty and the measurement uncertainty (that is, EDR) have been debated for years and generalized to be \(\Delta PAQ \geq \left|\langle\hat{P}, \hat{Q}\rangle\right| / 2\) and \(\epsilon(\hat{P}) \xi(\hat{Q}) \geq \left|\langle\hat{P}, \hat{Q}\rangle\right| / 2\), respectively. Although the former seems uncontroversial (6), which represents the fundamental limit on the measurement statistics for any state preparation, the latter was proven to be incorrect and can be violated experimentally (7). Following this observation, there has been a considerable amount of lively debate on uncertainty relations as a result of the new inequalities for generalizing original ones (8–14). Ozawa (8, 9), Hall (11), and Branciard (12) independently derived new inequalities for the EDR, which were later experimentally verified with polarized neutrons (15, 16) and photons (17–22).

The EDR implies the impossibility of simultaneously measuring two noncommuting variables to arbitrary precision. That is, a simultaneous measurement, called joint measurement, of \(\hat{P}\) and \(\hat{Q}\) indicates the capability of measuring \(\hat{P}\) without disturbing \(\hat{Q}\). Recently, Busch, Lahti, and Werner (BLW) have proposed an idea for joint measurements of qubits, by which a general error–trade-off relation is obtained as the uncertainty relation (23, 24). Because the joint measurement is available, one may approximate this joint measurement to the unavailable joint measurement of the other two operators, which follows the spirit of Heisenberg’s original idea in 1927, as claimed in the BLW proposal. Specifically, two compatible observables \(\hat{C}\) and \(\hat{D}\) are defined by Busch et al. (23, 24), which are noncommuting but own common eigenvectors. Because they can be measured jointly, \(\hat{C}\) and \(\hat{D}\) are used to approximate two incompatible observables \(\hat{A}\) and \(\hat{B}\). The BLW scheme aims to find combined approximation errors constrained by the incompatibility degree of the target observables, \(\hat{A}\) and \(\hat{B}\) (See Fig. 1 for a conceptual understanding of the idea.). The combined approximation errors are considered as the worst-case estimate of the inaccuracy, which are defined in the BLW proposal as figures of merit characterizing the performance of the measuring device, rather than the disturbance induced by the measurement. Meanwhile, different from the definition given in previous studies (8, 9, 11, 12), the BLW error–trade-off relation can be state-independent and provides a more reasonable bound of the measurement precision.

RESULTS

The system and the scheme

We report experimental verification of Heisenberg’s EDR by a single trapped \(^{40}\)Ca\(^{+}\) ion, following the BLW proposal. The atomic ion is confined in a harmonic trap, that is, within the Lamb-Dicke regime of a

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linear Paul trap, with an axial frequency of $\omega_z/2\pi = 1.01$ MHz and a radial frequency of $\omega_r/2\pi = 1.2$ MHz. We encode a qubit into two electronic levels $|\downarrow\rangle \equiv |S_{1/2}, m_J = -1/2\rangle$ and $|\uparrow\rangle \equiv |D_{3/2}, m_J = +3/2\rangle$, where $m_J$ is the magnetic quantum number (see Fig. 2A). Doppler cooling and resolved-sideband cooling are performed mainly along the axial direction, yielding the final average phonon number $\bar{n} < 0.1$ along the axial direction with the Lamb-Dicke parameter $\eta \sim 0.09$. Together with the optical pumping, the system is initially prepared in the ground state.

Before presenting our experimental results, we first specify some important points in our experimental scheme. We consider the positive operators $A_\pm = (I \pm \vec{a} \cdot \vec{\sigma})/2$ and $B_\pm = (I \pm \vec{b} \cdot \vec{\sigma})/2$ regarding $A$ and $B$, respectively, where $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ represents a vector associated with the usual Paul matrices. To be jointly measurable, the compatible observables $C$ and $D$, as the approximation of $A$ and $B$, own the positive operators $C_\pm = (I \pm \vec{c} \cdot \vec{\sigma})/2$ and $D_\pm = (I \pm \vec{d} \cdot \vec{\sigma})/2$, which satisfy $\|\vec{c}\| \leq 1$, $\|\vec{d}\| \leq 1$, and $\|\vec{c} + \vec{d}\| + \|\vec{c} - \vec{d}\| \leq 2$.

The essential step of our execution is the joint measurement $G$ on the compatible observables $C$ and $D$. In the study of Busch et al. (23), $G$ is associated with the POVM operators $G_{\pm,\pm}$ commonly owned by $C_\pm$ and $D_\pm$ with the marginality relation $C_\pm = G_{\pm,\pm} + G_{\pm,-}$ and $D_\pm = G_{\pm,\pm} + G_{-\pm,\pm}$. Generally speaking, the POVM can be achieved in a qubit with the assistance of another auxiliary qubit, implying the requirement of two qubits for implementing the operations. However, in our experiment, we construct the POVMs by single-qubit operations, and thus, the BLW scheme can be verified on a single qubit. To this end, the POVMs constructed are not general but satisfy the restricted condition

$$\text{rank}(G) \equiv 1 \quad (1)$$

which means that only some special POVMs are achievable in the single-qubit system. The condition also implies that we have to involve a prefactor $\text{Tr}[G]$ in the measurement of the POVM operator. Besides, in our ion trap system, the projective measurement is performed on $|\uparrow\rangle$. Thus, we have to first rotate the POVMs unitarily to be in line with $|\uparrow\rangle$ before making the measurements.

In a single-qubit system, for each POVM element operator $G_{\mu,\nu} (\mu, \nu = \pm)$ applied on a density operator $\rho$, the measurement result $p_p(G_{\mu,\nu})$ and the normalized density operator $K_p(G_{\mu,\nu})$ correspond, respectively, to $p_p(G_{\mu,\nu}) = \text{Tr}[G_{\mu,\nu}\rho]$ and $K_p(G_{\mu,\nu}) = \sqrt{G_{\mu,\nu}\rho}/\sqrt{G_{\mu,\nu}}$, with $\Sigma_{\mu,\nu}G_{\mu,\nu} = 1$. To simplify the description below, we rewrite $G_{\mu,\nu}$ as $G$ by neglecting the subscripts $\mu$ and $\nu$. Defining a pure-state measurement basis $|\psi\rangle$, we obtain a measurement $M$ along the same direction as $|\psi\rangle$ satisfying $M \equiv \text{Tr}[M|\psi\rangle\langle\psi|]$ and $\text{Tr}[M] = \text{Tr}[G]$. If there is a unitary operation $U$ mapping $G$ to $M = U G U^\dagger$, the density operator changes accordingly as $\rho' = U \rho U^\dagger$. Therefore, we reach important relations as below

$$p_p(G) = \text{Tr}(G)p_p(|\psi\rangle\langle\psi|)$$

$$K_p(G) = U K_p(|\psi\rangle\langle\psi|) U^\dagger$$

which are one-by-one mappings between the POVM and the projective measurement on $|\psi\rangle$. Because no unitary transformation changes the
rank of an operator, the POVM operator $G$ can be achieved by a pure-state–relevant positive operator, strictly obeying the condition in Eq. 1.

In our case with a single qubit consisting of the upper level $|\uparrow\rangle$ and the lower level $|\downarrow\rangle$, we assume that $G = g_0 I + \vec{g} \cdot \vec{\sigma}$. Provided that $\|\vec{g}\|^2 = g_0^2$ is satisfied and the ranks of all the POVM operators are units, the operators can be directly measured by combining a unitary operation and a projective measurement on $|\uparrow\rangle$. In addition, the marginality relations between $G_{A,B}$ and $G_{C,D}$ imply that the condition of $\|\vec{g}\|^2 = g_0^2$ is equivalent to $\|\vec{z} + \vec{d}\|^2 + \|\vec{z} - \vec{d}\|^2 = 2$, that is, $1 + \vec{z} \cdot \vec{d} = \|\vec{z} + \vec{d}\|$ and $1 - \vec{z} \cdot \vec{d} = \|\vec{z} - \vec{d}\|$, under which finding optimal approximations to $A$ and $B$ are always available (see the Supplementary Materials). In the trapped ion system, the unitary operators under the government of carrier transitions are accomplished by tuning the evolution time and the laser phase as explained in Fig. 2B. Thus, we obtain the Wasserstein distances (23) between $A$, $C$ and $B$, $D$, in association with Heisenberg’s EDR. Then, we examine the maximal uncertainty for various states of the system and different choices of $C$ and $D$.

In our implementation, we consider $A = \sigma_x$ with $A_z = (I + \sigma_z)/2$ and $B = \sigma_y$, with $B_z = (I + \sigma_z)/2$. As the approximation of $A$ and $B$, the two compatible observables $C$ and $D$ can be set as $C_z = (I + \alpha \sigma_y + \beta \sigma_z)/2$ and $D_z = (I - \alpha \sigma_y + \beta \sigma_z)/2$, where $\alpha^2 + \beta^2 = 1$ is satisfied as a result of the requirement for unit rank of the POVMs. In our case, $C_z$ and $D_z$ are not directly measurable but are obtained from the POVM operators $G_{A,C} = [I + \alpha \sigma_y + \beta \sigma_z]/4$ and $G_{B,C} = [I - \alpha \sigma_y + \beta \sigma_z]/4$. As clarified below, by using the carrier transition and then making projective measurements on $|\uparrow\rangle$, we can achieve measurements of the observables $A_{\downarrow}$, $B_{\downarrow}$, and $G_{A,C}$.

In the experiments presented below, we define $\alpha = \sin(\theta_1)$ and $\beta = \cos(\theta_1)$ (Fig. 1B). For a state $\rho$, the error measure (23, 25) between $A$ and $C$ is estimated by the Wasserstein distance $\Delta_{\rho}(A,C)^2$ (see Materials and Methods), and similarly, we have $\Delta_{\rho}(B,D)^2$ for the difference between $B$ and $D$. Heisenberg’s EDR for the pair of incompatible observables is determined by maximizing the summation of the two Wasserstein distances over all the possible states of the system with

$$\Delta_{\rho}(A,B)^2 := \max_{\rho} \Delta_{\rho}(A,C)^2 + \Delta_{\rho}(B,D)^2 = 2\sqrt{1 - \alpha^2} + (1 - \beta^2) \geq 2(\sqrt{2} - 1) (2)$$

where the second equality holds when the system is prepared in $|\psi\rangle = \cos(\theta_1/2)|\uparrow\rangle - i \sin(\theta_1/2)e^{i\phi}|\downarrow\rangle$, with $\theta_1$ and $\phi_1$ defined in Fig. 2B, and the state-independent lower bound $\Delta_{\rho}(A,B)^2 := 2(\sqrt{2} - 1)$ of the uncertainty is reached at $\alpha = \beta = 1/\sqrt{2}$. Equation 2 represents a worst-case estimate of the inaccuracy applicable to all possible states.

### Experimental observation

Under the rotating-wave approximation (see the Supplementary Materials), the Hamiltonian of our case in units of $\hbar = 1$ is given by $H_c = \Omega(\sigma_x e^{i\phi} + \sigma_z e^{i\phi})/2$ (26), where $\Omega$ is the Rabi frequency representing the laser-ion coupling strength, $\sigma_x$ are the usual Pauli operators, and $\phi$ is the laser phase. As shown in Fig. 2B, the experiment starts from the state $|\downarrow\rangle$, and the system evolves under $U_c(\theta, \phi)$, that is,

$$U_C(\theta, \phi) = \cos(\theta/2)I - i \sin(\theta/2)(\sigma_x \cos \phi - \sigma_y \sin \phi)$$

(3)

with $\theta = \Omega t$ determined by the evolution time.

To verify Heisenberg’s EDR, we vary $\alpha$ and $\beta$ to reach the maximal Wasserstein distance as in Eq. 2. The first step is to prepare the state $|\phi\rangle$. We fix the laser phase $\phi_1$ and steer the system under $U_c(\theta_1, \phi_1)$ toward $|\phi\rangle$, which is tuned with the change of $\alpha$ and $\beta$ for an optimal value corresponding to the worst-case estimate of inaccurate. The operation is executed by a 729-nm laser coupling $|\uparrow\rangle$ and $|\downarrow\rangle$ for 2 to 3 ms (see details in the Supplementary Materials). The second step is to measure the necessary observables $A_{\downarrow}$, $B_{\downarrow}$, and $G_{A,C}$, which is achieved by another evolution under $U_c(\theta_2, \phi_2)$ and then a detection on the state $|\uparrow\rangle$. To this end, we first drive the $|\downarrow\rangle \rightarrow |\uparrow\rangle$ transition by the 729-nm laser following the scheme in Table 1. Detection is then made by reapplying the cooling lasers and counting the emitted photons for 4 ms by the photomultiplier tube.

A faithful observation requires a clear understanding of the operational imperfections. From an effective period of Rabi oscillation, we estimate the error of the initial-state preparation to be $3(1/\%)$ and the imperfection in the detection to be 0.35(2/%) (the numbers in parentheses are the SEM). The radial thermal phonons cause a dephasing-like behavior that yields an accumulative deviation in the evolution. All these errors are experimentally determined, and the induced deviation can be corrected. Hence, the Rabi oscillation under a $\pi/2$ pulse of $U_c$ can reach a fidelity of 99.8(1/%) (see the Supplementary Materials), and thus, the observed data of $A$, $B$, $C$, and $D$ demonstrate an excellent agreement with the theoretical prediction. Errors, reflecting the fluctuation due to unstable laser power and magnetic field, are calculated and included in the SD.

Typical experimental data sets of $\langle A_z \rangle$, $\langle B_z \rangle$, $\langle C_z \rangle$, and $\langle D_z \rangle$ are depicted in Fig. 3, which clearly demonstrate no possibility of good approximations of $C$ to $A$ and $D$ to $B$, simultaneously. Provided $\theta_{in} \to \pi/2$, we have $C \to \sigma_y$, indicating the nearly perfect case for $C$ approaching $A$. However, in this case, we have $D \to I/2$, implying that we cannot obtain any information about $B$. With $\theta_{in}$ away from $\pi/2$, $D$ approaches $B$, and meanwhile, $C$ turns out to be much different from $A$. The sum of their differences, reflecting the balance between the two differences, reaches the minimum at $\theta_{in} = \pi/4$ (see the inset of Fig. 4).

The error–trade-off relation is witnessed in Fig. 4 by the Wasserstein distances $\Delta_{\rho}(A,C)^2$ and $\Delta(B,D)^2$, which are calculated by the experimental data in Fig. 3. The observation fits the theoretical prediction “when one is more precisely measured, the other is more disturbed” very well. One cannot predict both outcomes of two incompatible measurements to arbitrary precision. Because it results from the maximal Wasserstein distances over all the possible states in the system, the observed error–trade-off relation represents the state-independent inaccuracy and reflects the essence of Heisenberg’s EDR.

### Table 1. Scheme for the measurement pulses in experimental observation of the inaccuracy of the error–trade-off relation for $A = \sigma_y$ and $B = \sigma_x$.

<table>
<thead>
<tr>
<th>$A_z$</th>
<th>$A_z$</th>
<th>$B_z$</th>
<th>$B_z$</th>
<th>$G_{A,z}$</th>
<th>$G_{A,z}$</th>
<th>$G_{B,z}$</th>
<th>$G_{B,z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_2$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$0$</td>
<td>$\pi$</td>
<td>$2 \text{arccos} \left( \frac{1 + \beta}{2} \right)$</td>
<td>$2 \text{arccos} \left( \frac{1 - \beta}{2} \right)$</td>
<td>$2 \text{arccos} \left( \frac{1 + \beta}{2} \right)$</td>
</tr>
</tbody>
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Downloaded from http://advances.sciencemag.org on July 22, 2021.
Because $A$ and $B$ in our case are maximally incompatible, the lower bound of the uncertainty can reach $2(\sqrt{2} - 1)$, the minimum in Eq. 2. We plot this lower bound in Fig. 4 by the dashed line tangent to the state-independent curve of the error–trade-off relation. The tangent point implies $\alpha = \beta = 1/\sqrt{2}$. It is worth noting that the error bars, which dominantly resulted from the statistical deviation (due to quantum projection noise), represent the largest valid range of the experimental observation, rather than the true values allowed to be below the theoretically predicted lower bound of the uncertainty. Besides, the error bars here are four times as long as those in Fig. 3, reflecting the maximum possible deviation of statistics in measuring four variables (see Materials and Methods). More measurements can shrink the error bars but could not present new physics with respect to the 40,000 measurements (see Materials and Methods). More measurements can shrink the error bars but could not present new physics with respect to the 40,000 measurements. Moreover, by fixing $\theta$ and $\vartheta$ given in Table 1, whereas $\langle C_+ \rangle$ and $\langle D_+ \rangle$ are obtained from $\langle G_{28} \rangle$ by the marginality relation. The curves represent the results of theoretical prediction. The error bars indicate SD containing the statistical errors of 40,000 measurements for each data point as well as the errors from unstable laser power and fluctuating magnetic field.

**DISCUSSION**

Because modern technology has been progressing steadily toward the exploration of much smaller objects, our operations, particularly measurements, confront the ultimate quantum limits. As a result, Heisenberg’s uncertainty relation not only bounds the accuracy of the operations available with current laboratory techniques but also helps in understanding the very foundations of quantum mechanics. In quantum information science, the uncertainty relations have already been used to prove the security of quantum key distribution and the nonlocality. In addition, more in-depth research on the uncertainty relation may also bring new insights into the foundations of quantum mechanics. On the other hand, more in-depth research on the uncertainty relation may also bring new insights into the foundations of quantum mechanics, such as a deeper understanding of nonlocality.

We note that the BLW idea has stimulated broad interests in further exploring error–trade-off relations, such as the optimal joint measurement in a geometric manner and possible joint measurement for arbitrary observables of finite dimensional systems. Because the inequality, as the inaccuracy in the error–trade-off relation, is physically more specific than the measurement uncertainty or the preparation uncertainty, the BLW idea can be readily applied to further checking the security of quantum key distribution and the nonlocality. In addition, the inequality in the BLW scheme is different from other uncertainty relations. As a result, applying the BLW idea to quantum information science as done for other inequalities, for example, in the studies of Watanabe and Sagawa and Dressel and Nori, will help in scrutinizing the lowest bound among various uncertainty relations, which might optimize the available information gained on each qubit.

Our demonstration by a single ultracold trapped ion system is the first evidence to confirm the BLW-formulated error–trade-off relation in a pure quantum system. This is also an essential step toward...
understanding fundamental uncertainties of quantum mechanical
variables, the prerequisite of exploring limits of ultraprecision measure-
ments. Our experimental scheme is readily applicable to other trapped
ion species and single-spin systems for quantum information purposes.
The idea of achieving POVMs in a single-ion system will be applied to
other quantum tasks, such as accurately testing the inequalities in pre-
vious studies (32, 33) at the single-qubit level.

MATERIALS AND METHODS
Operation details
In our experiment, we demonstrated variation of the observables with
respect to α and β. Our operations included steering toward the state
|φ⟩ by $U_C(θ_1, φ_1)$ and realizing the observables by $U_C(θ_2, φ_2)$,
followed by the detection on the state |↑⟩. The step can be mathema-
tically written as

$$\langle K_L \rangle = \text{Tr}[K_L] \cdot \langle ↑ | U | ↓ \rangle | U ^ \dagger | ↑ \rangle$$

where $U = U_C(θ_2, φ_2) U_C(θ_1, φ_1)$ and $K = A, B, G$.

The trace distance for a pair of observables E and F, with $E_+ =
(ξ I + \vec{e} \cdot \vec{σ})/2 (E = I - E_+)$ and $F_+ = (ξ I + \vec{f} \cdot \vec{σ})/2 (F = I - F_+)$
applied on the state $\rho = (I + \vec{r} \cdot \vec{σ})/2$, is given by

$$\Delta_\rho (E, F)^2 := 2 \sum_{±} |p_±^E - p_±^F| = 2 |ξ_0 - ξ_1 + \vec{r} \cdot (\vec{e} - \vec{f})|$$

where $p_±^E = \text{Tr}[p_±| (I = E, F)$ is the probability distribution. In the qubit
case, the trace distance was actually the Wasserstein distance defined in
Busch et al. (23) for inaccuracies. Equation 5 shows that the SD of the
trace distance was four times that of the $p_0$ observed.

In our case, $\Delta_\rho (A, C)^2 = 2(1 - α) |r_2|$ and $\Delta_\rho (B, D)^2 = 2(1 -
β) |r_2|$, where $r_2 = \sin θ_1 \cos φ_1$ and $r_2 = - \cos θ_1$ (see the Supplementary
Materials). The probability distribution for the observables in the main

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