Toward the capacity limit of 2D planar Jones matrix with a single-layer metasurface

Yanjun Bao1*, Long Wen1, Qin Chen1, Cheng-Wei Qiu2*, Baojun Li1*

The Jones matrix is a useful tool to deal with polarization problems, and its number of degrees of freedom (DOFs) that can be manipulated represents its polarization-controlled capabilities. A metasurface is a planar structure that can control light in a desired manner, which, however, has a limited number of controlled DOFs (≤4) in the Jones matrix. Here, we propose a metasurface design strategy to construct a Jones matrix with six DOFs, approaching the upper-limit number of a 2D planar structure. We experimentally demonstrate several polarization functionalities that can only be achieved with high (five or six) DOFs of the Jones matrix, such as polarization elements with independent amplitude and phase tuning along its fast and slow axes, triple-channel complex-amplitude holography, and triple sets of printing-hologram integrations. Our work provides a platform to design arbitrary complex polarization elements, which paves the way to a broader exploitation of polarization optics.

INTRODUCTION
Polarization is one of the most important properties of electromagnetic waves (1). The optical elements for polarization control have become the cornerstone for a variety of optical applications and techniques. Technologically, the polarization manipulation by these optical elements can be represented by a 2 × 2 matrix, i.e., the Jones matrix. The Jones matrix has four components, each having two degrees of freedom (DOFs), i.e., the amplitude and phase terms. The introduction of the Jones matrix is very useful and convenient for problems concerning polarization manipulation. With the development of modern optics, there is an urgent need for different polarization elements with more advanced functionalities. Some of them can be realized by cascading several existing polarization elements, but most other functionalities are not. In addition, the cascading operation increases the volumes of optical devices and hinders minimization and integration. Generally, the capability of polarization control is associated with the number of the DOFs of the Jones matrix that can be manipulated, and therefore, reaching a higher number of DOFs of the Jones matrix is desired for more advanced functionalities.

A metasurface is a planar structure consisting of artificial optical scatterers that can arbitrarily control the amplitude, phase, and polarization of light at the subwavelength scale (2–8). A variety of Jones matrixes have been constructed by metasurfaces in the literature (8–26). The capability of the metasurface functionalities related to the number of DOFs of these Jones matrixes is summarized in Fig. 1A. The simplest case is the Jones matrix with one DOF, such as the 2π phase modulation in one of the four components (8–17), which can be used for metasurface hologram (11–14), focusing (15–17), etc. The case with two DOFs (e.g., independent amplitude and phase modulations of one component of the Jones matrix) has been used for more advanced functionalities (18–22), such as printing hologram integration (19–21). A typical example with three DOFs of the Jones matrix (23–25) is an optical element that can impose phase shifts φ11 and φ22 on light linearly polarized along its fast and slow axes rotated by an angle ϕ relative to the reference coordinate. Such an element has been demonstrated to allow independent phase control of arbitrary orthogonal states of polarization (23). Recently, we have shown the generation of light with arbitrary amplitude, phase, and polarization distribution (26), which has four parameters and therefore requires four DOFs in the Jones matrix (the expression with y-polarized incidence is shown in Fig. 1A). For a single-layer metasurface (SLM), because of its two-dimensional (2D) planar nature that has mirror symmetry with respect to its structural plane, the off-diagonal elements of the Jones matrix must be the same (27, 28). Therefore, only three components of the Jones matrix of SLM are independent, i.e., an upper-limit number of six DOFs. The construction of a Jones matrix with six DOFs indicates the ultimate and general control for 2D planar structures, which has not been realized previously.

Here, we aim to realize the goal of a Jones matrix with an upper limit of six DOFs for a 2D planar structure. We propose a design strategy of dielectric metasurface consisting of four nanoblock elements in one pixel, which can coherently contribute to the scattered optical fields. Each component of the Jones matrix is a function of the positions and orientational angles of the four nanoblocks and can be independently controlled by these geometric parameters, leading to a total of six DOFs of the Jones matrix. As a proof of concept, we experimentally demonstrate an optical element with five DOFs of the Jones matrix that can impose independent amplitude and phase control on its slow and fast axes with an arbitrary orientational angle. We further demonstrate triple amplitude-phase holograms and triple sets of printing and hologram integrations that are encoded in the three components of the Jones matrix, which is only attainable in a metasurface with an upper limit of six DOFs.

RESULTS
A schematic of the designed metasurface with an upper limit of six DOFs of the Jones matrix is depicted in Fig. 1 (B and C). The metasurface is a 2D periodical structure with periods of $P_x$ and $P_y$ along x and y directions, respectively. In each unit, we consider n crystal-silicon (c-silicon) nanoblocks (7) inside with the
coordinates and rotational angles denoted by \((x_k, \theta_k)\), where \(k\) is the index of the \(k\)th element. All the nanoblocks have the same sizes and are chosen with strong anisotropy that only responds to the incident polarization along its long axis. Assuming that a plane wave with polarization \(\mid \alpha_i \beta_i \rangle \) is incident from the substrate side at an angle of \(\sin(\lambda/P_x)\) relative to the \(z\) axis in the \(xz\) plane (\(\lambda\) denotes the incident wavelength), the transmitted diffraction along the normal directions \(E_i\) can be written as \(E_i \propto J \mid \alpha_i \beta_i \rangle\), where \(J\) is the Jones matrix with the following components (see the Supplementary Materials):

\[
J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \alpha_{11} e^{i\phi_{11}} & \alpha_{12} e^{i\phi_{12}} \\ \alpha_{21} e^{i\phi_{21}} & \alpha_{22} e^{i\phi_{22}} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} e^{2\pi i P_x x_k \cos^2 \theta_k} & \frac{1}{2} \sum_{k=1}^{n} e^{2\pi i P_x x_k \sin 2 \theta_k} \\ \frac{1}{2} \sum_{k=1}^{n} e^{2\pi i P_x x_k \sin 2 \theta_k} & \sum_{k=1}^{n} e^{2\pi i P_x x_k \cos^2 \theta_k} \end{bmatrix} \tag{1}
\]

As it shows, the components of the Jones matrix of the metasurface are constructed on the basis of the coherent scattering summations of the \(n\) nanoblocks in the pixel, which we therefore refer to as a coherent pixelated metasurface (7). Since the designed SLM has mirror symmetry with respect to the \(xy\) plane, the off-diagonal elements of the Jones matrix should be identical \(J_{21} = J_{12}\) (27, 28), which is verified in the above analytical equation. The construction of the Jones matrix with six DOFs indicates that the six parameters in Eq. 1 \((\alpha_{11}, \alpha_{12}/\alpha_{21}, \alpha_{22}/\alpha_{12}, \phi_{11}, \phi_{12}/\phi_{21}, \text{and } \phi_{22})\) can be arbitrarily valued. In Eq. 1, there are three complex equations, i.e., six real-number equations. For arbitrary given values of the six amplitude and phase parameters, the condition of existing solutions to the six equations requires at least six variables, i.e., a minimum number of three nanoblocks in one pixel (each nanoblock has two variables, \(x\) position coordinate and rotational angle). In practice, we found that the six variables may have no solutions for certain amplitude and phase parameters. Even for the cases with solutions, the values of \(x\) position coordinates of the three nanoblocks may be very close, leading to spatial overlapping between them. We therefore add an additional nanoblock into the pixel unit \((n = 4)\). For the spatial arrangement of these nanoblocks, every two nanoblocks are assigned with the same \(y\) coordinate and the four nanoblocks form two lines in total, as shown in Fig. 1C. The eight variables not only make the equation set solvable for arbitrary amplitude and phase parameters of the Jones matrix but also avoid spatial overlapping by adding additional constraints on the variables of \(x\) coordinates. The detailed restricted conditions in our design are \((x_2 - x_1) > P_x/4\) and \((x_4 - x_3) > P_x/4\), which guarantees that the coupling between nanoblocks can be negligible (see the Supplementary Materials). The six equations and the two inequality constraints are solved with genetic algorithm for a given set of amplitude and phase parameters (see details in fig. S4).
The algorithm is computationally fast (less than 1 s on a single-core personal computer) and takes about 2 hours for a typical metasurface with 100 × 100 pixels. Although the coherent design strategy is operated under oblique incidence, it is still effective for normal incidence if the full 2π phase shifts along the long or (and) short axis of the nanoblocks are considered (see the Supplementary Materials).

We emphasize that the Jones matrix proposed in this work is a general and ultimate form of SLM. All the Jones matrices of SLMs reported previously, such as the diatomic (26, 29, 30) or tetrameric (31) meta-atom designs, which have a maximal DOFs of four, fall into specific subsets of our design. Therefore, our strategy can not only be used to realize various functionalities of the previous SLMs with one unified design principle but also enable functional optical elements that are previously impossible, which will be demonstrated in the following.

The first demo we present is a fundamental but unreported optical element in metasurface area. As shown in Fig. 2A, the designed optical element imposes independent amplitude tunings $A_1$ and $A_2$ and phase shifts $\gamma_1$ and $\gamma_2$ on incident linearly polarized light along its fast and slow axes that is rotated with an angle of $\phi$ relative to the reference coordinate. The behavior of such an optical element looks like a conventional linearly birefringent wave plane, which has been realized with nanostructures such as plasmonic antennas (8) or dielectric pillars (9, 23, 32). However, compared with the previous strategies that can only tune the phase shifts, the proposed optical element introduces two additional DOFs, which are amplitude tuning $A_1$ and $A_2$. The Jones matrix of the designed element is

$$J = R(-\phi) \begin{bmatrix} A_1 e^{i\gamma_1}, & 0 \\ 0, & A_2 e^{i\gamma_2} \end{bmatrix} R(\phi)$$

where $R$ is a 2 × 2 rotation matrix. The Jones matrix of the element has five DOFs, but only three can be measured in the experiment, that is, $A_0 = A_1/A_2$, $\gamma = \gamma_1 - \gamma_2$, and $\phi$. Expanding Eq. 2, we obtain

$$J = A_2 e^{i(\gamma_1 + \gamma_2)/2} \left[ A_0 e^{i\gamma/2} \cos^2 \phi + e^{-i\gamma/2} \sin^2 \phi, \sin \phi \cos \phi (A_0 e^{i\gamma/2} - e^{-i\gamma/2}) \right] \sin \phi \cos \phi (A_0 e^{i\gamma/2} - e^{-i\gamma/2}), A_0 e^{i\gamma/2} \sin^2 \phi + e^{-i\gamma/2} \cos^2 \phi \right]$$

The term within the bracket on the right-hand side clearly shows the three independent parameters defined above and the same values of the off-diagonal elements. Note that Eq. 3 (five DOFs) is only a subclass of our designed Jones matrix in Eq. 1 (six DOFs). It can be verified that such a Jones matrix with six DOFs can impose independent amplitude and phase profiles on arbitrary orthogonal polarization states of input light.

Without loss of generality, we assume $A_0 = 2$, $\gamma = \pi/2$, and $\phi = \pi/3$ for an example to verify the design strategy. The sizes of the c-silicon nanoblocks are chosen with a width of 40 nm, a length of 170 nm, and a height of 600 nm, which have been verified with strong anisotropy that can only respond to the incident polarization along its long axis (26). A laser at wavelength 671 nm is used as the excitation source. The periods of the metasurface along $x$ and $y$ directions are $P_x = 900$ nm and $P_y = 450$ nm, respectively. We fabricated the metasurface structure on the c-silicon layer that is transferred on the glass substrate, using standard electron beam lithography. The details of the fabrication process are the same as that in our previous work (26). A scanning electron microscopy (SEM) image of the fabricated metasurface is shown in Fig. 2B.

Experimentally, we measured the transmission polarizations $|\alpha\beta|$ by recording the Stokes parameters (see Materials and Methods). The transmission polarization parameters are measured to be $\tan \alpha = 1.28$, $\beta = 5.55$ and $\tan \beta = 1.16$, $\beta = 1.49$ for $s$- and $p$-polarized incidences, respectively. We numerically fitted the measured results with the three parameters $A_0$, $\gamma$, and $\phi$ in Eq. 3 to reach a minimum value of a user-defined figure of merit $\eta$ (see Materials and Methods). The best-fitting parameters are obtained as $A_0 = 1.7529$, $\gamma = 1.675$, and $\phi = 0.993$ with $\eta$ value less than $1 \times 10^{-6}$. The good agreement between the measured and designed parameters (Fig. 2C) shows the correctness of our metasurface design for arbitrary 2D planar Jones matrix construction. The diffraction efficiency was measured to be 26.1% and 29.7% under $s$- and $p$-polarized incidences, respectively.

We consider the Jones matrix with full six DOFs to further verify the design strategy. We encode three independent complex-amplitude (i.e., amplitude and phase) holographic images “A,” “B,” and “C” into the three components of the Jones matrix of the metasurface, which requires six DOFs (each component needs two). Phase-only holograms are also designed for comparison. For the phase-only hologram, its amplitude distribution is fixed uniform at maximal value, and its phase profile is obtained with the iterative Gerchberg-Saxton algorithm (33). For the complex-amplitude hologram, the amplitude and phase of the hologram in each component of the Jones matrix is directly obtained with a single inverse Fresnel transform of the input holographic image. A SEM image of the fabricated metasurface
is shown in Fig. 3A, where the coherent pixels with different geometric parameters can be observed.

To measure the holographic images, the incident light is tuned as $s$ or $p$ polarization and the transmitted scattering light by the metasurface is filtered with $x$ or $y$ polarization (see Materials and Methods). The different combinations of the incident and output polarizations result in four cases, corresponding to the four components of the Jones matrix of metasurface. The two different types of holographic images measured through the independent three channels are illustrated in Fig. 3B, reproducing the designed image patterns well. Because of the speckle noise for the phase-only hologram, the qualities of the reconstructed holographic images are much lower than those of the complex-amplitude hologram. A detailed calculation shows that the root mean square error (RMSE) of the measured phase-only images with respect to the designed ones (maximal intensity set as 1) is about two times or more than that of the amplitude-phase one (Fig. 3C). For the complex-amplitude hologram, we define an efficiency that is independent on the metasurface area (see the Supplementary Materials), which is measured as 5.9%, 8.3%, and 7.0% for the holographic images of letters “A,” “B,” and “C,” respectively. In comparison, the corresponding hologram efficiencies of the phase-only cases are 11.2%, 15.1%, and 13.8%, respectively.

Recently, the simultaneous amplitude and phase control has been used for the integration of printing and holographic images (19–21, 34–36). Therefore, our design approach can encode three sets of printing and holographic images into the three components of the Jones matrix of the single metasurface (Fig. 4A), with the set number more than ever reported before (usually one set). Before the design procedure, there are two important things that need to be noted in the integration of the two types of images. First, the amplitude and phase of the metasurface are not constant but vary for different pixels. If the adjacent pixels are too close to each other, their interferences with different phases may lead to the disorder of the printing image. This problem can be solved by designing an enlarged pixel constructed by several subpixels with the same phase (20) or simply increasing the distance between pixels. The distributions of the nanostructures are more dispersed in the latter solution and its holography efficiency is lower than the former one. Since the efficiency is not our objective, we adopt a simple method (the latter solution) to give a proof-of-concept demonstration and increase the pixel pitch to 4 $\mu$m (Fig. 4B). Second, the input printing image should be chosen with intensity as high as possible, which is beneficial to increase the holography efficiency. This is because the intensity of the printing image determines the amplitude of the metasurface and therefore the holography efficiency.

We choose three emoji images with bright background as the input printing images (Fig. 4A). The holographic images are designed as letter images of “A,” “B,” and “C,” the same as in Fig. 3. Figure 4C shows our measured printing and holographic images encoded in the three components of the Jones matrix of the metasurface, which exhibit the designed image patterns well. One can observe that there are some impurities with bright spots emerging in the printing images with $J_{11}$ and $J_{22}$ channels but totally disappearing in the ones with the $J_{12}/J_{21}$ channels. Such phenomenon can be understood by the fact that the output detection polarizations are almost the same with (or totally orthogonal to) the input ones for $J_{11}/J_{22}$ ($J_{12}/J_{21}$).

**Fig. 3.** Triple complex-amplitude holography enabled by the Jones matrix with six DOFs. (A) SEM image of the coherent pixelated metasurface for triple complex-amplitude holographic images. The dashed rectangle indicates one pixelated unit. The periods are $P_x = 900$ nm and $P_y = 450$ nm. (B) Measured holographic images encoded in the three components of the Jones matrix ($J_{11}$, top; $J_{12}/J_{21}$, middle; $J_{22}$, bottom) with phase-only (left column) and amplitude-phase (right column) hologram design. The holographic images are located at 500 $\mu$m above the metasurface plane. (C) Comparison of the RMSE of the measured holographic images between the phase-only (blue bars) and amplitude-phase (red bars) hologram design. The maximal intensity of the images is assumed to be 1.
multilayer metasurface design to break the mirror symmetry, which increases the DOFs of the Jones matrix. Further increasing the DOFs to eight requires the tuning of the positions and orientational angles of the nanoblocks. Optical elements with five DOFs and printing-hologram functions with six DOFs of the Jones matrix are experimentally demonstrated. Because of the mirror symmetry of the planar nature of the metasurface, the Jones matrix of SLM has an upper-limit of six DOFs. Further increasing the DOFs to eight requires multilayer metasurface design to break the mirror symmetry, which is the future research direction. Our strategy using a coherent pixelated metasurface for the Jones matrix with six DOFs, which is the maximum number realized to date, lays the foundation of designing complex optical polarization elements and is expected to have various applications in polarization optics and holography.

MATERIALS AND METHODS

Optical measurement

The 671-nm laser is incident obliquely in the xz plane. The incident polarization is controlled by a linear polarizer (LP). The scattered light by the metasurface is collected by an objective (10×/0.25) and filtered with another LP, which is tuned to select the x or y polarization to pass through. A tube lens is used to focus the holographic image on a complementary metal-oxide semiconductor color camera. When measuring the printing images in Fig. 4, the objective is replaced by a 4×/0.1 one.

Parameter measurement of the optical element in Fig. 2

The polarizations of the transmission in terms of the Stokes parameters are measured with an LP and a quarter-wave plate (QWP). The intensities for different pairs of the rotational angles of LP and QWP are measured by a power meter (37). Under s- and p-polarized incidences, the polarizations of transmission are measured as $\left| a_i^p, b_i^p \right|$ and $\left| a_i^s, b_i^s \right| = \left[ \cos \alpha_i, \sin \alpha_i \right]^T$ and $\left[ \cos \beta_i, \sin \beta_i \right]^T$, respectively. The analytical polarizations of a Jones matrix with $J_{ij}$ ($i,j=1,2$) components as shown in Eq. 3 under s- and p-polarized incidences are $\left| a_i^p, b_i^p \right| = \left[ J_{11}, J_{12} \right]^T \sqrt{j_{12}^2 + j_{22}^2}$ and $\left| a_i^s, b_i^s \right| = \left[ J_{11}, J_{12} \right]^T \sqrt{j_{11}^2 + j_{21}^2}$, respectively. The parameters $A_0$, $\gamma$, and $\Phi$ are fitted to the experimental data with a minimal value of figure of merit defined as $\eta = 2 - \left( \left| a_i^s, b_i^s \right| \right)^2 - \left( \left| a_i^p, b_i^p \right| \right)^2$. For the perfect-fitting case, $\eta = 0$.

DISCUSSION

In summary, we have proposed a metasurface design strategy with its Jones matrix approaching the upper-limit number of six DOFs for a 2D planar structure, which represents the ultimate ability of polarization control that an SLM can have. By combining four nanoblocks in one pixel and considering their coherent scattering properties, we reveal how to construct such a Jones matrix with the tuning of the positions and orientational angles of the nanoblocks. Optical elements with five DOFs and printing-hologram functionalities with six DOFs of the Jones matrix are experimentally demonstrated. Because of the mirror symmetry of the planar nature of the metasurface, the Jones matrix of SLM has an upper-limit of six DOFs. Further increasing the DOFs to eight requires multilayer metasurface design to break the mirror symmetry, which


Acknowledgments

Funding: This work was supported by the National Natural Science Foundation of China (62075246, 11804407, 61827822, 92050108, and 11874029) and the Guangdong Natural Science Foundation (2018A030313333). C.-W.Q. acknowledges the financial support by NUSRI via grant R-2018-5-001 and the support from the National Research Foundation, Prime Minister’s Office, Singapore under its Competitive Research Program (CRP award NRF CRP22-2019-0006). Author contributions: Y.B. conceived the idea and performed the simulation, sample fabrication, and experiment. L.W. and Q.C. assisted with the experiments. All authors discussed the results and commented on the manuscript. Competing interests: The authors declare that they have no competing interests. Data and materials availability: All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials.

Submitted 12 February 2021
Accepted 5 May 2021
Published 18 June 2021
10.1126/sciadv.abh0365

Citation: Y. Bao, L. Wen, Q. Chen, C.-W. Qiu, B. Li, Toward the capacity limit of 2D planar Jones matrix with a single-layer metasurface. Sci. Adv. 7, eabh0365 (2021).
Toward the capacity limit of 2D planar Jones matrix with a single-layer metasurface
Yanjun Bao, Long Wen, Qin Chen, Cheng-Wei Qiu and Baojun Li

Sci Adv 7 (25), eabh0365.
DOI: 10.1126/sciadv.abh0365