Observation of second sound in a rapidly varying temperature field in Ge

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Second sound is known as the thermal transport regime where heat is carried by temperature waves. Its experimental observation was previously restricted to a small number of materials, usually in rather narrow temperature windows. We show that it is possible to overcome these limitations by driving the system with a rapidly varying temperature field. High-frequency second sound is demonstrated in bulk natural Ge between 7 K and room temperature by studying the phase lag of the thermal response under a harmonic high-frequency external thermal excitation and addressing the relaxation time and the propagation velocity of the heat waves. These results provide a route to investigate the potential of wave-like heat transport in almost any material, opening opportunities to control heat through its oscillatory nature.

INTRODUCTION

The study of heat transport beyond Fourier’s regime has attracted renewed interest in recent years. Great efforts have been performed to unravel the physical properties of thermal waves, as well as the experimental conditions that are necessary for their observation. Applications based on such concepts have been envisioned and discussed extensively already in many recent publications (1–4). The spatiotemporal propagation of the temperature field in the form of waves is known as “second sound,” a term that was adopted in analogy to “first sound” (or simply “sound,” i.e., mechanical lattice vibrations). As pointed out in (5), first sound and second sound are described by a similar equation where the variables have a different physical meaning, i.e., pressure and temperature, respectively.

The simplest differential equation that describes wave-like heat transport from a mesoscopic perspective is the hyperbolic heat equation (HHE) according to Maxwell, Cattaneo, and Vernotte

\[
\tau_{ss} \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} - \alpha \nabla^2 T = \frac{1}{\rho C_p} \left( S(r, t) + \tau_n \frac{\partial S(r, t)}{\partial t} \right)
\]

(1)

where \(\alpha\) is the thermal diffusivity, \(\tau_{ss}\) is the thermal relaxation time, \(\rho\) is the mass density, \(C_p\) is the specific heat, and \(S(r, t)\) is an external power heat source. In our context, the system is in local equilibrium, and it is well characterized by a local temperature \(T\) (see discussion in section S8). The previous equation describes the propagation of a temperature wave with a damping term given by \(\partial T/\partial t\) and a propagation velocity \(v_{ss} = \sqrt{\alpha/\tau_{ss}}\). The solutions of this equation lead to different heat transport regimes, depending on the temporal and spatial length scales under investigation. The key to unlock the different regimes is the magnitude of the first term on the left-hand side of Eq. 1, thermal inertial term; i.e., if \(\tau_{ss} \gg \partial^2 T/\partial t^2\) is sufficiently large, the spatiotemporal distribution of temperature field will exhibit wave-like behavior.

Second sound in solids was first experimentally observed in solid He (6); later in NaF (7), Bi (8), and SrTiO₃ (9); and most recently in highly oriented pyrolytic graphite (10). Several theoretical works have also recently addressed its occurrence in low-dimensional systems (11–13). In all these experimental observations of second sound, the dominance of momentum conserving phonon scattering (Normal processes) with respect to resistive phonon scattering (Umklapp processes) was found to be the key mechanism leading to its observation. Second sound was observed almost exclusively in the very low temperature regime (\(T < 5\) K), with the exception of a recent example (10) at higher temperatures (125 K) for samples with low resistive phonon scattering.

A condition for the experimental detection of second sound, based on these experimental observations (6–10), was found to be \(\tau_N < \tau_{exp} < \tau_R\), i.e., the typical experimental observation times (\(\tau_{exp}\)) must be larger than normal phonon scattering times (\(\tau_N\)) to allow momentum redistribution but smaller than resistive phonon scattering times (\(\tau_R\)) to avoid decay of the phonon wave packet into the phonon equilibrium distribution. The theoretical foundations of second sound were set in the 1960s by M. Chester, R. J. Hardy, C. P. Enz, and co-workers (5, 14, 15), who predicted the existence of “drifting” second sound, i.e., a type of wave-like heat transport that is triggered by the dominance of normal phonon scattering events. This type of second sound was experimentally confirmed in (6–10). Furthermore, the existence of a “driftless” or “high-frequency” type, as well as “other types” of second sound, was also envisioned. In this case, the dominance of normal scattering events is not a necessary condition for the existence of wave-like heat transport, but the key requirement is instead the slow decay of the energy flux, as predicted by different solutions to the Boltzmann transport equation (5). These different flavors of second sound are all contemplated in the mesoscopic HHE (Eq. 1), because different microscopic mechanisms can lead to the propagation of thermal waves.

We show that it is possible to observe a type of high-frequency second sound in natural bulk Ge by driving the system out of equilibrium with a rapidly varying temperature field. Our concept is based on taking advantage of the second-order time derivative in the HHE, Eq. 1, in a frequency-domain experiment. As the driving frequency increases toward the hundreds of megahertz range, the

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relative weight of this term with respect to the damping term (first-order time derivative) increases proportionally to the frequency upon a harmonic excitation, hence making the observation of wave-like heat propagation possible. We show that this approach is robust enough to expose second sound independently, to a certain extent, of the phonon scattering rates of the studied material, as well as of temperature. Although heat transport in Ge is dominated by resistive phonon scattering processes, which partly originate from its large isotopic diversity, we show that it is still possible to observe second sound in the high-frequency limit.

RESULTS AND DISCUSSION

Our experiments are based on a frequency-domain optical reflectance pump-and-probe approach based on two lasers with different wavelengths ($\lambda_{\text{pump}} = 405$ nm and $\lambda_{\text{probe}} = 532$ nm) focused onto the surface of a Ge sample to a spot size with radius $R_{\text{spot}} \approx 5.5 \mu$m. The studied samples are pieces of a substrate of natural Ge. Further details are provided in the Supplementary Materials (section S1). The pump laser (thermal excitation) is modulated between 30 kHz and 200 MHz with a sinusoidal power output waveform, leading to a dynamic modulation of the optical reflectivity of the surface of the sample, which is also well described by a harmonic waveform. A frequency-dependent phase lag gradually develops, defined as a function of frequency (section S3). Whereas the lattice contribution to the optical reflectivity is at least 20-fold larger than the electronic component at the higher frequencies, for lower frequencies, this ratio is substantially increased.

Figure 1B displays the experimental phase lag as a function of frequency between 30 kHz and 200 MHz at room temperature. The complex thermal response of the specimen was, at first, computed numerically within Fourier’s model, solving the parabolic approximation to the three-dimensional (3D) HHE (diffusive case), which is obtained when the first term of Eq. 1 can be neglected. In the range between 30 kHz and 1 MHz, the agreement between Fourier’s solution and the experimental data is excellent, although deviations are already observed around 1 MHz. Above 30 MHz, the difference between the experimental phase lag and Fourier’s predictions is evident. For the higher-frequency range, the experimental data show that the phase lag (absolute value) decreases with increasing frequency. This trend cannot even be qualitatively reproduced by Fourier’s model, which predicts that as frequency increases, the phase lag approaches $-\pi/4$ and even lower values (see section S7 and fig. S7-3). The full 3D solution of the HHE based on the finite-element method (FEM) was used to fit the experimental data through the entire frequency range, and it is shown in Fig. 1B. A detailed description of the fitting procedure is presented in section S7. We obtained $\delta_{\text{exp}} = 500$ ps and $\alpha_{\text{exp}} = 3 \times 10^{-5}$ m$^2$/s, thus leading to a propagation velocity $v_{\text{exp}} = 250$ m/s, all at room temperature. These experimental observations are in qualitative agreement with the computational experiments (see schematic illustration in Fig. 1C) by nonequilibrium molecular dynamics (NEMD) and are shown in the inset of Fig. 1B (see also section S5 and fig. S5-1). Although a quantitative agreement cannot be expected because of differences between the experimental and the computational setups (reduced size of the sample and purely 1D heat flux in NEMD), the NEMD results show a notable similarity with the experimental results, with...
the penetration depth of the wave-like and diffusive contributions is (full set in section S2). As temperature decreases, the ratio between are similar. The temperature dependence of \( \tau \) and \( \tau_{\text{ss}} \) (section S7), as well as the diffusive and wave-like limits, \( \tau_{\text{diff}} = \sqrt{\alpha / (\pi \tau_{\text{eq}})} \) and \( \tau_{\text{ss}} = 2 \sqrt{\alpha \tau_{\text{ss}}} \), respectively. A critical frequency \( f_c \) is obtained when \( \Lambda_{\text{diff}} = \Lambda_{\text{ss}} \), thus providing an estimation of the frequency for which the diffusive and wave-like contributions to heat transport are similar. The temperature dependence of \( \tau_{\text{ss}} \), \( \tau_{\text{eq}} \), and \( f_c \) was studied between room temperature and 7 K, and it is shown in Fig. 2A (full set in section S2). As temperature decreases, the ratio between the penetration depth of the wave-like and diffusive contributions is \( \Lambda_{\text{ss}} / \Lambda_{\text{diff}} = \sqrt{4 \pi / \tau_{\text{ss}}(T)} \), which implies that lower temperatures favor the spatial propagation of the thermal waves because larger \( \tau_{\text{ss}} \) is expected and indeed experimentally observed for lower temperatures. Wave-like effects are already present below \( f_c \), as can be observed comparing the experimental data with the corresponding fits using the HHE to the Fourier predictions, as shown in Fig. 2A. The onset frequency can be estimated as \( f > 0.1 f_c \), corresponding to a phase lag difference between the experimental data and the Fourier solution >\( 5^\circ \). The onset of wave-like effects is also evidenced by the deviations between \( \Lambda_{\text{diff}} \) and \( \Lambda_{\text{HHE}} \) as shown in Fig. 1D. The minimum observed on the phase lag curves and the position of the critical frequency \( f_c \) relative to the frequency of the minimum originate from the relation between \( \alpha \), \( \tau_{\text{ss}} \), and \( R_{\text{pot}} \) (see discussion in section S7). The frequency-dependent phase lag in Fig. 2A was fitted at each temperature using the HHE, as described for the room temperature case, and the results for \( \tau_{\text{ss}} \) and \( \tau_{\text{eq}} \) are shown in Fig. 2B and table S2. We note that the accuracy of the fits reduces for lower temperatures due to the frequency-dependent temperature rise induced by the pump laser.

To understand the origin of these observations, we have developed a rather simple model (see derivation in section S6) based on the expansion of the perturbed phonon distribution function \( f_{\text{ph}} \), i.e., the intermediate state assumed by the nonequilibrium distribution before it decays to the equilibrium one by means of dissipative resistive processes, as: \( f_{\text{ph}} = f_{\text{eq}} + \beta_{\lambda} \cdot q + \gamma_{\lambda} \cdot (\partial q / \partial t) \), where \( f_{\text{eq}} \) is the equilibrium phonon distribution function, \( q \) is the heat flux, \( \beta_{\lambda} \) and \( \gamma_{\lambda} \) are mode-dependent functions to be determined, \( t \) is the temporal coordinate, and \( \lambda \) denotes each phonon mode. We note that the case with \( \gamma_{\lambda} = 0 \) leads to the same propagation velocity for second sound as in (5) (section S6). In our case, however, the presence of rapidly varying temperatures leads to rapidly varying \( q \), suggesting that the expansion of the perturbed phonon distribution...
function in terms of $\partial q/\partial t$ is a reasonable assumption. The previous ansatz for $f$ was then introduced into the linearized Boltzmann transport equation (BTE) to find the solution for $\beta_2$ and $\gamma$. It can be shown that within this framework, it is possible to derive the HHE, with a corresponding explicit expression for $\tau_{ss}$ in terms of individual phonon relaxation times ($\tau_\lambda$) as shown in section S6

$$
\tau_{ss} = \sum_\lambda \hbar \omega_\lambda v_\lambda^2 \frac{2}{\hbar} \frac{\partial f_{eq}}{\partial T} \frac{1}{\tau_{ss}}
$$

where $\hbar$ is the Planck constant, $\omega_\lambda$ is the phonon energy, and $v_\lambda$ is the phonon group velocity. We computed $\omega_\lambda$, $\tau_\lambda$, and $v_\lambda$ from the solution of the BTE based on density functional theory (DFT) interatomic force constants (IFCs) (section S4 and table S4). We restricted ourselves to the relaxation time approximation (RTA) after verifying that the full iterative BTE picture does not alter the prediction of the theory. Using the RTA has the additional benefit of providing a comparison on equal footing with previous theoretical descriptions (5) and allowing unambiguous definition of the relaxation times (24). We observe that the corrections to the RTA provided by a full iterative solution of the BTE are very small (within 4% in the thermal conductivity) for Ge at temperatures as low as 50 K (fig. S4-1). The resulting values were inserted into Eq. 2, which yielded $\tau_{ss}^{\text{theo}}$ and $\tau_{ss}^{\text{theo}} = \sqrt{\alpha^{\text{theo}}/\tau_{ss}}$ as a function of temperature, as shown in Fig. 2B. The agreement of the predicted values with those obtained from the experiments is remarkable for $T > 100$ K, considering that the values of $\omega_\lambda$, $\tau_\lambda$, and $v_\lambda$ are evaluated within a fully ab initio scheme. We note that the model leading to Eq. 2 is expected to be valid for $|\Delta T| < T$, where $|\Delta T|$ is the amplitude of the laser-induced thermal oscillations and $T$ is the absolute temperature as set by the cryostat. In our experiments, $|\Delta T|$ max < 20 K; thus, the observed deviations between the theoretical predictions and the measured values at very low temperatures are expected (see section S8 for details). We note that similar experiments (25, 26) were explained in terms of nonlocality as described by the Guyer-Krumhansl equation, which reduces to Eq. 1 in the absence of nonlocal effects. The present experiments, however, are beyond the applicability of this equation because the heating region is much smaller than the nonlocal length (27). Future work should address the attenuation of nonlocal effects under the present conditions and its influence in the propagation of thermal waves.

The spatial dependence of the temperature field in the parabolic (diffusive) and the hyperbolic (wave-like) cases was simulated using FEMs in the direction perpendicular to the surface of the sample at an arbitrary time. Figure 2C displays the normalized temperature profiles for 15, 100, and 300 K at the highest experimental excitation frequency of ≈300 MHz (see fig. S7-5 for simulations of the temperature field at $f_\lambda$). As expected, the wave-like behavior of the temperature field exhibits a strong temperature dependence. The observed propagation depth, particularly at lower temperatures, is especially interesting if considering the possibility of high-frequency modulated thermal interference. We think that the present approach could open interesting possibilities for the experimental observation of wave-like heat transport in other materials and lead to the development of strategies to control heat transport.

MATERIALS AND METHODS

Samples

All samples were pieces cleaved from a 2-inch-diameter nominally undoped Ge wafer with (100) crystallographic orientation, high resistivity (>40 ohm-cm), and etch pit density <3000/cm². The wafer was purchased from International Wafer Service Inc. (USA). We have studied three different types of samples: (i) clean Ge (no native oxide), (ii) Ge + native oxide + 60 nm of Au, and (iii) a 60-nm-thick...
Au transducer was evaporated onto the surface of a Ge piece similar to (i). Additional details on the etching and evaporation processes of the samples are given in section S1.

**Amplified frequency-domain thermoreflectance**

We developed a low-noise custom-built frequency-domain thermoreflectance setup to measure the thermal response of the samples. The thermal excitation was provided by a pump laser diode with a wavelength of 405 nm. The probe laser used was a continuous-wave (CW) laser of 532-nm wavelength and with a maximum output power of 100 mW. The output power was controlled with neutral density filters to ~50 µW (CW) for the probe and ~20 mW (RMS) for the pump laser. A 30-mm achromatic lens was used to focus both Gaussian beams onto the same spot, whose size was measured using the knife-edge method to a 1/e² radius of ~5.5 µm. A portion of the probe reflected light was sent to an avalanche photodiode detector. Two notch filters were inserted into the optical path with the purpose of individually blocking the laser components. First, the probe component was blocked, and the phase of the pump laser was measured using a high-frequency lock-in amplifier. After performing this phase calibration step, the notch filter mount was mechanically displaced to block the pump laser component, thus allowing us to measure the harmonic signal arising from the probe laser. An extended description of the experimental setup is given in section S2.

**Finite-element modeling**

The 3D electron diffusion-recombination equation is solved using FEMs with COMSOL Multiphysics to estimate the electronic contribution to the photoreflectance signal (see section S3C). The 3D HHE is also solved using FEM to calculate the phase lag between the harmonic laser excitation and the temperature response of the system (see section S7). The laser energy deposition is restricted to a region defined by the Gaussian function of the pump beam in the radial direction and an exponential decay in the cross-plane direction with the characteristic length of the optical penetration. The temperature oscillations correspond to a weighted average across the surface of the sample computed using the Gaussian function of the probe beam as the weight.

**Ab initio calculations**

The thermal conductivity and the heat flux relaxation time expressions in terms of the velocities and the relaxation times of the phonon modes are obtained by solving the BTE (see derivation in section S6). The required microscopic properties are calculated within DFT using VASP code (28). The harmonic IFCs were calculated from finite differences in a 5 × 5 × 5 supercell, while we used a 4 × 4 × 4 supercell for the anharmonic ones, limiting the interactions to fourth neighbors. The inequivalent displacements needed to obtain the IFCs were obtained with the phonopy (29) and thirddorder.py (24) codes (more details can be found in section S4).

**Nonequilibrium molecular dynamics**

Computational experiments within NEMD used a 5 × 5 × 145 supercell of the eight-atom cubic Ge conventional cell. The interatomic interactions were described by a bond-order potential of the Tersoff type (30). All the NEMD simulations were performed using the LAMMPS code (31). The equations of motion were integrated with a time step of 1 fs, and temperature control was obtained by Nose-Hoover thermostating, while equations of motions have been integrated by the velocity-Verlet algorithm (more details can be found in section S5).

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/7/27/eabg4677/DC1

**REFERENCES AND NOTES**

19. Note that there is a temperature offset of about 20 K between low and high frequencies. The origin of this offset is possibly bandgap renormalization arising from the larger DC system response at high frequency.


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