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Supplementary Materials for

A quantum Fredkin gate

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The PDF file includes:

S1. Erasing the which-path information

Generation of path-mode entanglement, and successful operation of the gate in the quantum regime, relies on the erasure of the which-path information in the two displaced Sagnac interferometers. We test this by performing a Hong-Ou-Mandel (HOM) two-photon interference measurement after each interferometer. After overlapping path modes 2*R* and 1*G* on an NPBS a HWP with its OA set to 22.5° rotates the polarisation of the photons to $|D\rangle$ and $|A\rangle$, respectively. Sending these photons into the same port of a PBS leads to bunching at the output if the path-modes are indistinguishable. Doing the same for modes 2*B* and 1*Y* gives two separate HOM dips with visibilities 90±5% and 91±6%. In the main article, the HOM dips were measured simultaneously for each displaced Sagnac interferometer where path modes 2*R* and 1*G*, and 2*B* and 1*Y* are overlapped on a non-polarising beamsplitter (NPBS). The temporal delay in arrival times of the photons was varied by scanning the position of the input coupler for path modes 1*G* and 1*Y*. The variation in accumulated four-fold events as a function of temporal delay results in a dip with a visibility defined as $V = (C_{max} - C_{min})/C_{max}$, where C_{max} and C_{min} are the maximum and minimum number of four-fold events, respectively. In Fig S1 we measure *V*=90±5% for modes 2*R* and 1*G* and *V*=91±6% for modes 2*B* and 1*Y*, confirming a high degree of indistinguishability.

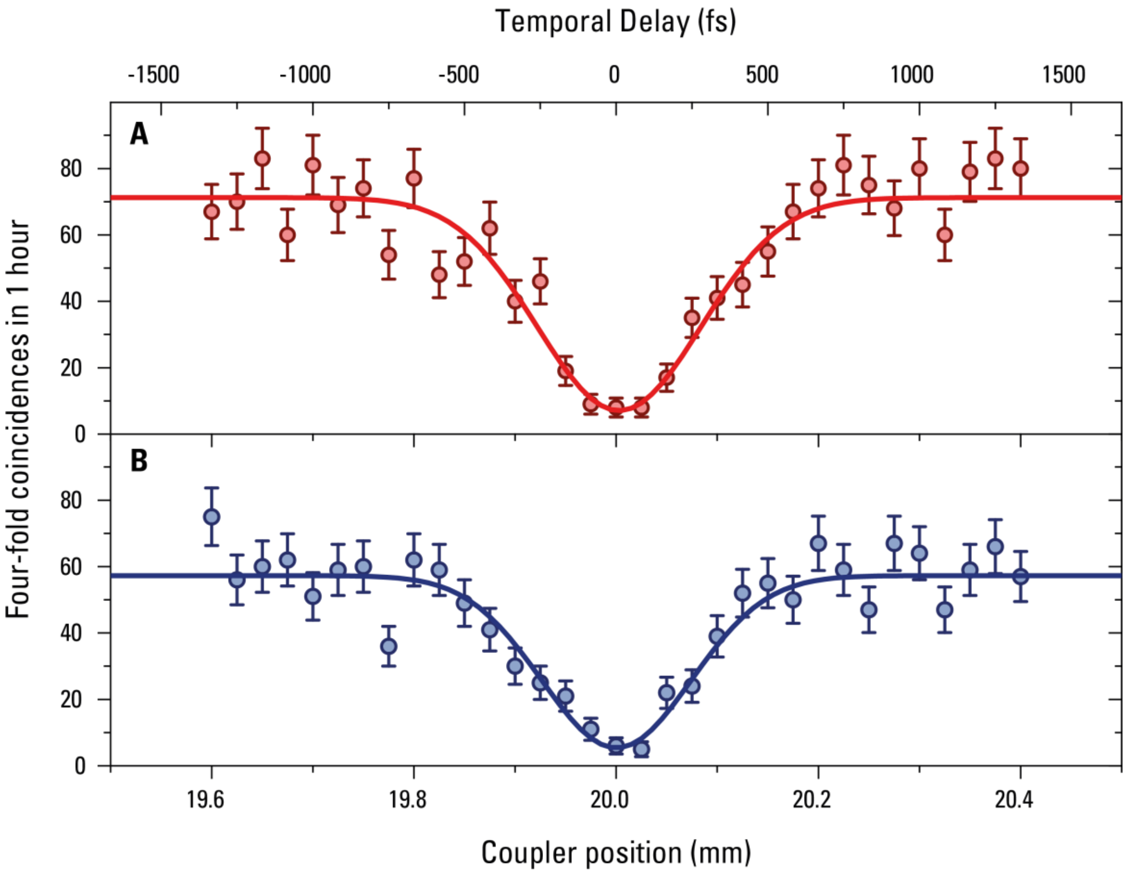

fig. S1. HOM dip measurements testing the indistinguishability of path-modes after each displaced Sagnac interferometer. **(A)**, Overlap of 2*R* and 1*G* with V=90±5%, and **(B)**, overlap of 2*B* and 1*Y* with V=91±6% .

S2. Generation of three-photon GHZ states

Here we outline how our quantum Fredkin gate generates four of the eight maximally entangled three-photon GHZ states. The general four-photon state prior to state preparation is

$$\left(|\xi\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|H\rangle_{2G}^{Tr}+|\xi\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|V\rangle_{2Y}^{Tr}\right)/\sqrt{2} \tag{S1}$$

The photons in modes 2*G* and 2*Y* pass through a HWP with its OA set to 22.5° and then impinge on a PBS leading to

$$\begin{aligned}&\frac{|\xi\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|H\rangle_{2G}^{Tr}}{\sqrt{4}}+\frac{|\xi\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|V\rangle_{2G}^{Tr}}{\sqrt{4}}+\\&\frac{|\xi\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|H\rangle_{2Y}^{Tr}}{\sqrt{4}}-\frac{|\xi\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|V\rangle_{2Y}^{Tr}}{\sqrt{4}}\end{aligned} \tag{S2}$$

In order to prepare the gate in a superposition of the SWAP and identity operations the control qubit is set to $|D\rangle^{C}=\frac{1}{\sqrt{2}}\left(|H\rangle^{C}+|V\rangle^{C}\right)$ giving

$$\begin{aligned}&\frac{|H\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|H\rangle_{2G}^{Tr}}{\sqrt{8}}+\frac{|V\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|H\rangle_{2G}^{Tr}}{\sqrt{8}}\\&+\frac{|H\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|V\rangle_{2G}^{Tr}}{\sqrt{8}}+\frac{|V\rangle_{1B}^{C}|\psi\rangle_{2B}^{T1}|\varphi\rangle_{1G}^{T2}|V\rangle_{2G}^{Tr}}{\sqrt{8}}\\&+\frac{|H\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|H\rangle_{2Y}^{Tr}}{\sqrt{8}}+\frac{|V\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|H\rangle_{2Y}^{Tr}}{\sqrt{8}}\\&-\frac{|H\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|V\rangle_{2Y}^{Tr}}{\sqrt{8}}-\frac{|V\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|V\rangle_{2Y}^{Tr}}{\sqrt{8}}\end{aligned} \tag{S3}$$

In the control arm of our experiment, the interferometer is arranged such that terms with $|H\rangle_{1B}^{C}$ or $|V\rangle_{1R}^{C}$ in equation (S3) are rejected. Rejecting these terms and swapping modes 2*B* and 1*G* gives

$$\begin{aligned}&\frac{|H\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|H\rangle_{2Y}^{Tr}}{\sqrt{4}}+\frac{|V\rangle_{1B}^{C}|\varphi\rangle_{1G}^{T1}|\psi\rangle_{2B}^{T2}|H\rangle_{2G}^{Tr}}{\sqrt{4}}\\&-\frac{|H\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}|V\rangle_{2Y}^{Tr}}{\sqrt{4}}+\frac{|V\rangle_{1B}^{C}|\varphi\rangle_{1G}^{T1}|\psi\rangle_{2B}^{T2}|V\rangle_{2G}^{Tr}}{\sqrt{4}}\end{aligned} \tag{S4}$$

Consequently from equation (S4) detection of a trigger photon in state $|H\rangle$ or $|V\rangle$ will result in two states with a relative phase difference of π.

$$|H\rangle^{Tr}:\frac{|H\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}}{\sqrt{2}}+\frac{|V\rangle_{1B}^{C}|\varphi\rangle_{1G}^{T1}|\psi\rangle_{2B}^{T2}}{\sqrt{2}} \tag{S5}$$

$$|V\rangle^{Tr}: \frac{|H\rangle_{1R}^{C}|\psi\rangle_{2R}^{T1}|\varphi\rangle_{1Y}^{T2}}{\sqrt{4}} - \frac{|V\rangle_{1B}^{C}|\varphi\rangle_{1G}^{T1}|\psi\rangle_{2B}^{T2}}{\sqrt{2}} \tag{S6}$$

As we detect both polarisations of the trigger photon, it is necessary to perform a classical phase rotation to the coincidence data corresponding to the detection of $\left|V\right\rangle^{Tr}$. The two sets of coincidence data are then combined. After erasing the which-path information, setting $\left|\psi\right\rangle = \left|V\right\rangle$ and $\left|\varphi\right\rangle = \left|H\right\rangle$ and recalling that $\left|H\right\rangle = \left|1\right\rangle$ and $\left|V\right\rangle = \left|0\right\rangle$ we obtain the three-photon GHZ state

$$\left|GHZ_1^+\right\rangle = \frac{\left(\left|0\right\rangle^C\left|1\right\rangle^{T1}\left|0\right\rangle^{T2} + \left|1\right\rangle^C\left|0\right\rangle^{T1}\left|1\right\rangle^{T2}\right)}{\sqrt{2}} \tag{S7}$$

Alternatively, setting $\left|\psi\right\rangle = \left|H\right\rangle$ and $\left|\varphi\right\rangle = \left|V\right\rangle$ produces

$$\left|GHZ_2^+\right\rangle = \frac{\left(\left|0\right\rangle^C\left|0\right\rangle^{T1}\left|1\right\rangle^{T2} + \left|1\right\rangle^C\left|1\right\rangle^{T1}\left|0\right\rangle^{T2}\right)}{\sqrt{2}} \tag{S8}$$

Preparing the control in $\left|A\right\rangle^C = \frac{1}{\sqrt{2}}\left(\left|H\right\rangle^C - \left|V\right\rangle^C\right)$ and setting $\left|\psi\right\rangle = \left|V\right\rangle$ and $\left|\varphi\right\rangle = \left|H\right\rangle$ gives

$$\left|GHZ_1^-\right\rangle = \frac{\left(\left|0\right\rangle^C\left|1\right\rangle^{T1}\left|0\right\rangle^{T2} - \left|1\right\rangle^C\left|0\right\rangle^{T1}\left|1\right\rangle^{T2}\right)}{\sqrt{2}} \tag{S9}$$

while for $\left|\psi\right\rangle = \left|H\right\rangle$ and $\left|\varphi\right\rangle = \left|V\right\rangle$

$$\left|GHZ_2^-\right\rangle = \frac{\left(\left|0\right\rangle^C\left|0\right\rangle^{T1}\left|1\right\rangle^{T2} - \left|1\right\rangle^C\left|1\right\rangle^{T1}\left|0\right\rangle^{T2}\right)}{\sqrt{2}} \tag{S10}$$