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Supplementary Materials for

Topological bootstrap: Fractionalization from Kondo coupling

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Derivation of Effective Spin Hamiltonians for Chiral Spin Liquid

The Model

System A consists of spinful fermions ($c_{i\alpha}$) at half filling on the sites i of the kagome lattice, and system B consists of spin-$1/2$s (S_i) on the kagome sites. The Hamiltonian is

$$H = H_A + H_B + H_{AB} \quad (1)$$

$$H_A = H_\uparrow(\nu = 1) + H_\downarrow(\nu = 1) \quad (2)$$

$$H_B = 0 \quad (3)$$

$$H_{AB} = g \sum_i c_i^\dagger \vec{\sigma} c_i \cdot \vec{S}_i \quad (4)$$

where $H(\nu = 1)$ is constructed following [24].

In particular, H_A involves nearest neighbor hopping $t = 1$ of spin up and down electrons with a background ϕ flux per triangle. There is some freedom in choosing the parameter ϕ, and we take $\phi = \pi/2$. (The system has gapless Dirac cones for $\phi = 0, \pi$). This choice yields a fully gapped band structure with ground state Chern number 1.

Effective Two Spin Interactions

We begin by calculating the coefficient for the induced nearest neighbor Heisenberg interaction ($\vec{S}_1 \cdot \vec{S}_2$). For simplicity, we will calculate the coefficient of $S_1^z S_2^z$. This is given by

$$-g^2 \sum_{E \neq E_0} \frac{\langle \psi_0 | c_{1\uparrow}^\dagger c_{1\uparrow} - c_{1\downarrow}^\dagger c_{1\downarrow} | \psi_E \rangle \langle \psi_E | c_{2\uparrow}^\dagger c_{2\uparrow} - c_{2\downarrow}^\dagger c_{2\downarrow} | \psi_0 \rangle}{E - E_0} + (1 \leftrightarrow 2) \tag{5}$$

The intermediate state can only involve an excitation in one spin sector. Using the symmetry between up and down spins, and the fact that conjugating the expression interchanges 1 and 2, we get

$$-4g^2 \sum_{E \neq E_0} Re \frac{\langle \psi_0 | c_{1\uparrow}^\dagger c_{1\uparrow} | \psi_E \rangle \langle \psi_E | c_{2\uparrow}^\dagger c_{2\uparrow} | \psi_0 \rangle}{E - E_0} \tag{6}$$

Using $c_i = (1/\sqrt{N}) \overline{\sum}_k c_k e^{ik \cdot R_i}$, where N is the number of Bravais lattice sites (or equivalently, the size of the Brillouin zone mesh), we see that $c_i^\dagger c_i = (1/N) \sum_{k,k'} c_k^\dagger c_{k'}$ (R_i can be set to 0 if sites $1, 2$ are in the same unit cell).

Hence, the coefficient reduces to

$$\frac{-4g^2}{N^2} \sum_{k,k',n,n'} Re \frac{\langle \psi_0 | c_{1\uparrow}^\dagger c_{1\uparrow} | k, n \to k', n' \rangle \langle k, n \to k', n' | c_{2\uparrow}^\dagger c_{2\uparrow} | \psi_0 \rangle}{E_{k',n'} - E_{k,0}} \tag{7}$$

where the excited state denotes removing an electron from band $n \in (1, 2, 3)$ at k and putting it in band $n' \in (4, 5, 6)$ at k' (There are six bands because the unit cell has been enlarged; see [24]).

Discretizing over a 10×10 mesh, with $t = 1$ for the hopping norm, the coefficient is $0.045g^2$(see fig. S1 for convergence).

Using the same procedure, the second and third neighbor interaction coefficients are $0.023g^2$ and $0.025g^2$.

Effective Three Spin Interactions

We now calculate the coefficient of $S_0^z S_1^x S_2^y$ of the spin chirality term. This is given by

$$g^3 \sum_{E_1, E_2 \neq E_0} \frac{\langle \psi_0 | c_{0\uparrow}^\dagger c_{0\uparrow} - c_{0\downarrow}^\dagger c_{0\downarrow} | \psi_{E_1} \rangle \langle \psi_{E_1} | c_{1\uparrow}^\dagger c_{1\downarrow} + c_{1\downarrow}^\dagger c_{1\uparrow} | \psi_{E_2} \rangle \langle \psi_{E_2} | - i c_{2\uparrow}^\dagger c_{2\downarrow} + i c_{2\downarrow}^\dagger c_{2\uparrow} | \psi_0 \rangle}{(E_1 - E_0)(E_2 - E_0)} \quad (8)$$
$$+(perm.0,1,2)$$

Again, since $0,1,2$ is related to $2,1,0$ by complex conjugation, and since cyclic permutations give the same result (model has C_3 rotational symmetry), we get

$$\frac{12 g^3}{N^3} \sum_{E_1, E_2 \neq E_0} Re \frac{i \langle \psi_0 | c_0^\dagger c_0 | E_2 \rangle \langle E_2 | c_1^\dagger c_1 | E_1 \rangle \langle E_1 | c_2^\dagger c_2 | \psi_0 \rangle}{(E_1 - E_0)(E_2 - E_0)} \quad (9)$$

E_1 must be an excitation transferring an electron from k, n ($n \in (1,2,3)$) to k', n' ($n' \in (4,5,6)$). E_2 can either be a further particle transfer on top of E_1: k', n' ($n \in (4,5,6)$) to k'', n'' ($n'' \in (4,5,6)$) or a hole transfer on top of E_1: k'', n'' ($n'' \in (1,2,3)$) to k, n ($n \in (1,2,3)$). Numerically, we have to beware of double counting the case $E_1 = E_2$. Our numerics give $-0.022 g^3$ (see fig. S2 for convergence).

The analysis of the triangular lattice model mentioned in the main text was completed using the same techniques.

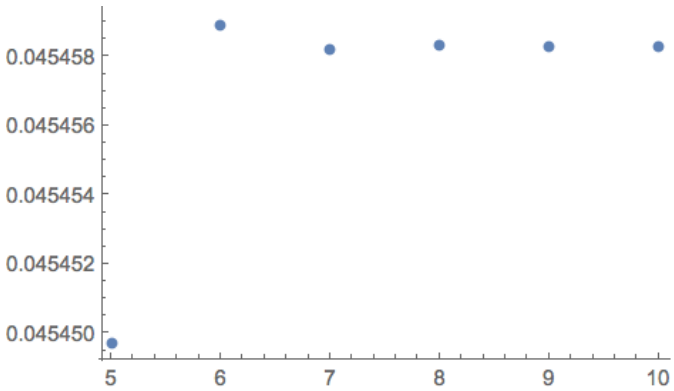

fig. S1. Convergence in two-spin interaction coefficient as a function of mesh linear dimension.

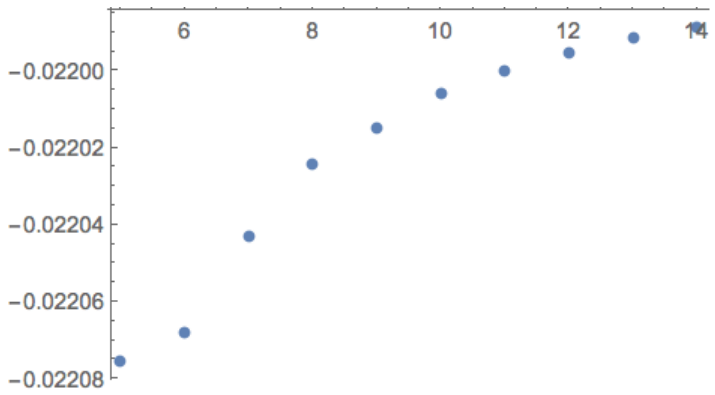

fig. S2. Convergence in spin chirality strength as function of mesh linear dimension.