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Supplementary Materials for

Photothermally induced transparency

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Section S1. Modeling

We investigate the dynamics of our system using the equations of motion in the rotating frame of control frequency ω_{con}

$$\dot{x}_{\text{th}} = -\gamma_{\text{th}}(x_{\text{th}} + \beta P_c) \tag{1}$$

$$\dot{a} = -[\kappa/2 - i(\Delta + Gx_{\text{th}})]a + \varepsilon_{\text{con}} + \varepsilon_p e^{-i\Omega_p t} \tag{2}$$

where x_{th} is the total cavity length change due to photothermal effects, γ_{th} is the photothermal relaxation rate, and $\beta = |dx_{\text{th}}/dP_c|$ is the photothermal expansion coefficient. The sign of β here is negative due to the outwards expansion of the front cavity mirror and the refractive index change of the its substrate. The control field (amplitude $\varepsilon_{\text{con}} = \sqrt{P_{\text{con}}\kappa_{\text{f}}/\hbar\omega_{\text{cav}}}$) is detuned from the cavity resonance by $\Delta = \omega_{\text{con}} - \omega_{\text{cav}}$, and the frequency of the probe (amplitude $\varepsilon_p = \sqrt{P_p\kappa_{\text{f}}/\hbar\omega_{\text{cav}}}$) is $\Omega_p = \omega_p - \omega_{\text{con}}$ in the rotating frame of the control frequency. The total loss rate of the cavity, κ, includes an external loss rate and an intrinsic loss rate. The intracavity power is $P_c = \hbar\omega_{\text{cav}}|a|^2/\tau_{\text{cav}}$, where $\tau_{\text{cav}} = 2L_c/c$ is the cavity round-trip time and L_c is the cavity length. The cavity mode and the photothermal effect are coupled at the rate $G = \omega_{\text{cav}}/L_c$.

We can linearize Eqs. (1)-(2) using the assumptions that $x_{\text{th}} = x_0 + \delta x_{\text{th}}$, $a = a_0 + \delta a$, and $a^* = a_0^* + \delta a^*$ in the case that the probe field is much weaker than the control field. We obtain following steady state solutions after doing the linearization

$$x_0 = -\alpha|a_0|^2 \tag{3}$$

$$a_0 = \frac{\varepsilon_{\text{con}}}{\kappa/2 - i(\Delta + Gx_0)} \tag{4}$$

where $\alpha = \beta\hbar\omega_{\text{cav}}/\tau_{\text{cav}}$, and the linearized dynamical equations

$$\delta\dot{x}_{\text{th}} = -\gamma_{\text{th}}[\delta x_{\text{th}} + \alpha(a_0\delta a^* + a_0^*\delta a)] \tag{5}$$

$$\delta\dot{a} = -\kappa\delta a/2 + i\Delta_0\delta a + iGa_0\delta x_{\text{th}} + \varepsilon_p e^{-i\Omega_p t} \tag{6}$$

where $\Delta_0 = \Delta + Gx_0$ is the effective detuning of the control laser from the cavity resonance. Considering the following ansatzs

$$\delta x_{th} = qe^{-i\Omega_{\rm p}t} + q^* e^{i\Omega_{\rm p}t} \tag{7}$$

$$\delta a = A^- e^{-i\Omega_{\rm p}t} + A^+ e^{i\Omega_{\rm p}t} \tag{8}$$

$$\delta a^* = (A^-)^* e^{i\Omega_{\rm p}t} + (A^+)^* e^{-i\Omega_{\rm p}t} \tag{9}$$

we obtain

$$(i\Omega_{\rm p} - \gamma_{\rm th})q = \gamma_{\rm th}\alpha[a_0(A^+)^* + a_0^*(A^-)] \tag{10}$$

$$(-i(\Delta_0 + \Omega_{\rm p}) + \kappa/2)A^- = iGa_0 q + \varepsilon_{\rm p} \tag{11}$$

$$(i(\Delta_0 - \Omega_{\rm p}) + \kappa/2)(A^+)^* = -iGa_0^* q \tag{12}$$

We can easily get the solution as

$$A^- = \frac{1 + if(\Omega_{\rm p})}{[-i(\Delta_0 + \Omega_{\rm p}) + \kappa/2] + 2\Delta_0 f(\Omega_{\rm p})}\varepsilon_{\rm p} \tag{13}$$

with

$$f(\Omega_{\rm p}) = \frac{G\gamma_{th}\alpha\,|a_0|^2}{[i(\Delta_0 - \Omega_{\rm p}) + \kappa/2](i\Omega_{\rm p} - \gamma_{\rm th})} \tag{14}$$

When there is no photothermal interaction, cavity length does not change, i.e., $q = 0$. From Eq. (12), we get $A^+ = 0$ in this case, implying that there is only one sideband inside the cavity which is the intracavity probe field.

We thus have the cavity reflection and transmission as follows

$$
\begin{aligned}
r &= S_{in} - \kappa a/2 \\
&= \varepsilon_{con} + \varepsilon_p e^{-i\Omega_p t} - \frac{\kappa}{2}(a_0 + A^- e^{-i\Omega_p t} + A^+ e^{i\Omega_p t}) \\
&= \varepsilon_{con} - \frac{\kappa}{2}a_0 + (\varepsilon_p - \frac{\kappa}{2}A^-)e^{-i\Omega_p t} - \frac{\kappa}{2}A^+ e^{i\Omega_p t} \qquad (15) \\
|r|^2 &= [\varepsilon_{con} - \frac{\kappa}{2}a_0 + (\varepsilon_p - \frac{\kappa}{2}A^-)e^{-i\Omega_p t} - \frac{\kappa}{2}A^+ e^{i\Omega_p t}][\varepsilon_{con} - \frac{\kappa}{2}a_0^* + (\varepsilon_p - \frac{\kappa}{2}(A^-)^*)e^{i\Omega_p t} - \frac{\kappa}{2}(A^+)^* e^{-i\Omega_p t}] \\
&= (\varepsilon_{con} - \frac{\kappa}{2}a_0)(\varepsilon_{con} - \frac{\kappa}{2}a_0^*) + (\varepsilon_p - \frac{\kappa}{2}A^-)(\varepsilon_p - \frac{\kappa}{2}(A^-)^*) + \frac{\kappa}{2}A^+(A^+)^* \\
&\quad +[(\varepsilon_{con} - \frac{\kappa}{2}a_0^*)(\varepsilon_p - \frac{\kappa}{2}A^-) - (\varepsilon_{con} - \frac{\kappa}{2}a_0)\frac{\kappa}{2}(A^+)^*]e^{-i\Omega_p t} \\
&\quad +[(\varepsilon_{con} - \frac{\kappa}{2}a_0)(\varepsilon_p - \frac{\kappa}{2}A^-)^* - (\varepsilon_{con} - \frac{\kappa}{2}a_0^*)\frac{\kappa}{2}A^+]e^{i\Omega_p t} \\
&\quad -(\varepsilon_p - \frac{\kappa}{2}A^-)\frac{\kappa}{2}(A^+)^* e^{-2i\Omega_p t} - (\varepsilon_p - \frac{\kappa}{2}A^-)^* \frac{\kappa}{2}A^+ e^{2i\Omega_p t} \qquad (16) \\
t &= \kappa a/2 \\
&= \frac{\kappa}{2}(a_0 + A^- e^{-i\Omega_p t} + A^+ e^{i\Omega_p t}) \qquad (17) \\
|t|^2 &= \frac{\kappa^2}{4}(a_0 + A^- e^{-i\Omega_p t} + A^+ e^{i\Omega_p t})[a_0^* + (A^-)^* e^{i\Omega_p t} + (A^+)^* e^{-i\Omega_p t}] \\
&= \frac{\kappa^2}{4}[a_0 a_0^* + A^-(A^-)^* + A^+(A^+)^* \\
&\quad +(a_0^* A^- + a_0(A^+)^*)e^{-i\Omega_p t} + (a_0(A^-)^* + a_0^* A^+)e^{i\Omega_p t} \\
&\quad +A^-(A^+)^* e^{-2i\Omega_p t} + A^+(A^-)^* e^{2i\Omega_p t}] \qquad (18)
\end{aligned}
$$

Here, we assume that the total cavity loss κ is equally contributed by the two cavity mirrors. Given that $a_0 \gg A$, the normalized oscillation part (T) of the transmitted signal is easily obtained by ignoring the higher-order terms

$$
T = \left| \frac{\kappa a_0^* A^- + \kappa a_0 (A^+)^*}{2 a_0 \varepsilon_p} \right| \cos(\Omega_p t + \phi_T) \qquad (19)
$$

where ϕ_T indicates the phase behavior of the cavity transmission.

Section S2. Shape of the transparency window of the cavity transmission

We can see that the transmission windows in Fig 3 a,b, and Fig 4b follow a Lorentzian shape, in this section we will derive an expression for this. We begin with the amplitude of cavity transmission

$$T_{amp} = \left| \frac{\kappa a_0^* A^- + \kappa a_0 (A^+)^*}{2 a_0 \varepsilon_p} \right| \tag{20}$$

The square of this may be expanded as

$$\begin{aligned} |T_{amp}|^2 = &\left[\kappa^2 \left(\Omega_p^2 + \gamma_{th}^2\right)\left(4(\Delta_0 - \Omega_p)^2 + \kappa^2\right)\right] / \\ &\left[64\alpha^2 |a_0|^2 \Delta_0^2 G^2 \gamma_{th}^2 - 16\alpha |a_0|^2 \Delta_0 G \gamma_{th} \left(4\Delta_0^2 \gamma_{th} + \kappa^2 \gamma_{th} - 4\Omega_p^2 (\kappa + \gamma_{th})\right) \right. \\ &\left. + \left(\Omega_p^2 + \gamma_{th}^2\right)\left(4(\Delta_0 - \Omega_p)^2 + \kappa^2\right)\left(4(\Delta_0 + \Omega_p)^2 + \kappa^2\right)\right] \end{aligned} \tag{21}$$

We wish to approximate $|T_{amp}^2|$ over the transmission window, which occurs when Ω_p is of the same order as γ_{th}. To analyse this regime we introduce dimensionless parameters

$$\tilde{\Delta}_0 = \frac{\Delta_0}{\kappa}; \ \tilde{\Omega}_p = \frac{\Omega_p}{\kappa}; \ \tilde{G} = \frac{G\alpha}{\kappa}; \ \kappa' = \frac{\kappa}{\gamma_{th}} \tag{22}$$

In terms of these we have

$$\begin{aligned} |T_{amp}|^2 = &\left[(1 + 4(\tilde{\Delta}_0 - \tilde{\Omega}_p)^2)(1 + (\kappa' \tilde{\Omega}_p)^2)\right] / \\ &\left[64 |a_0|^2 \tilde{G}^2 \tilde{\Delta}_0^2 + 16 |a_0|^2 \tilde{G} \tilde{\Delta}_0 (-1 - 4\tilde{\Delta}_0^2 + 4(1 + \kappa')\tilde{\Omega}_p^2) \right. \\ &\left. + (1 + 4(\tilde{\Delta}_0 - \tilde{\Omega}_p)^2)(1 + (\kappa' \tilde{\Omega}_p)^2)(1 + 4(\tilde{\Delta}_0 + \tilde{\Omega}_p)^2)\right] \end{aligned} \tag{23}$$

We now approximate $|\tilde{\Omega}_p| \ll |\tilde{\Delta}_0|$, as our detuning Δ_0 is of order κ while Ω_p is of order γ_{th}. This corresponds to sending

$$\begin{aligned} \tilde{\Delta}_0 \pm \tilde{\Omega}_p &\to \tilde{\Delta}_0 \\ -4\tilde{\Delta}_0^2 + 4(1 + \kappa')\tilde{\Omega}_p^2 &\to -4\tilde{\Delta}_0^2 \end{aligned} \tag{24}$$

in which case the transmission window becomes (returning to the original variables)

$$T_{window} = \frac{\kappa^2 (4\Delta_0^2 + \kappa^2)(\gamma_{th}^2 + \Omega_p^2)}{\gamma_{th}^2 \left(4\Delta_0(\Delta_0 - 2|a_0|^2 G\alpha) + \kappa^2\right)^2 + (4\Delta_0^2 + \kappa^2)^2 \Omega_p^2} \tag{25}$$

We can recover the Lorentzian shape by subtracting (25) from the response of a bare cavity evaluated at $\Omega_p = 0$

$$\frac{1}{1+\left(\frac{\Delta_0}{\kappa/2}\right)^2} - \frac{\kappa^2(4\Delta_0^2+\kappa^2)(\gamma_{th}^2+\Omega_p^2)}{\gamma_{th}^2\left(4\Delta_0(\Delta_0-2|a_0|^2G\alpha)+\kappa^2\right)^2+(4\Delta_0^2+\kappa^2)^2\Omega_p^2} \tag{26}$$

$$= \frac{16\alpha|a_0|^2\Delta_0 G\kappa^2\gamma_{th}^2\left(4\alpha|a_0|^2\Delta_0 G - 4\Delta_0^2-\kappa^2\right)}{\gamma_{th}^2\left(4\Delta_0^2+\kappa^2\right)\left(4\Delta_0(\Delta_0-2\alpha|a_0|^2G)+\kappa^2\right)^2+\omega^2\left(4\Delta_0^2+\kappa^2\right)^3} \tag{27}$$

Section S3. Calibration of probe transmission

If focusing on the behavior of the probe field, we will only look at the sideband of the frequency which is the same as the probe. From Eq. (17), the normalized probe transmission is obtained as

$$\begin{aligned} t_p &= \frac{\kappa A^-}{2\varepsilon_p} \\ &= \frac{[1+if(\Omega_p)]\kappa/2}{[-i(\Delta_0+\Omega_p)+\kappa/2]+2\Delta_0 f(\Omega_p)} \end{aligned} \tag{28}$$

Experimentally, the amplitude and phase of the probe transmission are calibrated from the measurement of the amplitude and phase of cavity transmission T.

Combining Eq. (10)-(13), we can also get the solution of q and $(A^+)^*$ as follows

$$q = \frac{[-i(\Delta_0+\Omega_\mathrm{p})+\kappa/2]A^- - \varepsilon_\mathrm{p}}{iGa_0} \tag{29}$$

$$(A^+)^* = \frac{-iGa_0^* q}{i(\Delta_0-\Omega_\mathrm{p})+\kappa/2} \tag{30}$$

According to Eq. (13) and Eq. (30), we have the following relation between A^- and $(A^+)^*$

$$(A^+)^* = \eta A^- \tag{31}$$

where

$$\eta = (\eta_2 - \eta_1)\eta_3 \tag{32}$$

$$\eta_1 = \frac{[-i(\Delta_0 + \Omega_\mathrm{p}) + \kappa/2] + 2\Delta_0 f(\Omega_\mathrm{p})}{1 + if(\Omega_\mathrm{p})} \tag{33}$$

$$\eta_2 = -i(\Delta_0 + \Omega_\mathrm{p}) + \kappa/2 \tag{34}$$

$$\eta_3 = \frac{-iGa_0^*}{i(\Delta_0 - \Omega_\mathrm{p}) + \kappa/2} \tag{35}$$

We can therefore calibrate the amplitude (t_p^{amp}) and phase (t_p^{phase}) of probe transmission based upon the measured cavity transmission T using the following equations

$$t_p^{amp} = \left|\frac{a_0}{a_0^* + a_0\eta}\right| T^{amp} \tag{36}$$

$$t_p^{phase} = \arg\frac{\mathrm{a}_0}{\mathrm{a}_0^* + \mathrm{a}_0\eta} + \phi_\mathrm{T} \tag{37}$$

where the T^{amp} is the amplitude of T.

Section S4. A simplified solution

If we assume $(A^+)^* \approx 0$, we have a simplified version of the solution. Though this approximation might not be valid in our case, the solution can still help us understand how the photothermal effects influence the system. Equations (10)-(12) are reduced to

$$(i\Omega_\mathrm{p} - \gamma_\mathrm{th})q = \gamma_\mathrm{th}\alpha a_0^* A^- \tag{38}$$

$$(-i(\Delta_0 + \Omega_\mathrm{p}) + \kappa/2)A^- = iGa_0 q + \varepsilon_\mathrm{p} \tag{39}$$

We obtain the solution as

$$\begin{aligned} A^- &= \frac{\varepsilon_\mathrm{p}}{(-i(\Delta_0 + \Omega_\mathrm{p}) + \kappa/2) - \frac{iG\varepsilon\alpha|a_0|^2}{(i\Omega_\mathrm{p} - \gamma_\mathrm{th})}} \\ &= \frac{\varepsilon_\mathrm{p}}{-i(\Delta_0 + \Omega_\mathrm{p} - \frac{\gamma_\mathrm{th}^2 G\alpha|a_0|^2}{\Omega_\mathrm{p}^2 + \gamma_\mathrm{th}^2}) + (\kappa/2 - \frac{G\varepsilon\alpha|a_0|^2\Omega_\mathrm{p}}{\Omega_\mathrm{p}^2 + \gamma_\mathrm{th}^2})} \end{aligned} \tag{40}$$

From the equation above, we can find the cavity detuning is shifited by $-\frac{\gamma_{\text{th}}^2 G\alpha|a_0|^2}{\Omega_{\text{p}}^2+\gamma_{\text{th}}^2}$ and the cavity decay is modified by the photothermal effects by

$$\kappa_{th} = -\frac{G\varepsilon\alpha\,|a_0|^2\,\Omega_{\text{p}}}{\Omega_{\text{p}}^2+\gamma_{\text{th}}^2} \tag{41}$$

When Ω_{p} approaches zeros, we have a window of Lorentzian shape which can be described by the normalized reflectivity as follows

$$A^- = \frac{\varepsilon_{\text{p}}(i\Omega_{\text{p}}-\gamma_{\text{th}})}{\kappa(i\Omega_{\text{p}}-\gamma_{\text{th}})/2 - iG\gamma_{\text{th}}\alpha\,|a_0|^2} \tag{42}$$

$$\begin{aligned} r &= 1-\frac{k(i\Omega_{\text{p}}-\gamma_{\text{th}})/2}{\kappa(i\Omega_{\text{p}}-\gamma_{\text{th}})/2 - iG\gamma_{\text{th}}\alpha\,|a_0|^2} \\ &= \frac{-iG\gamma_{\text{th}}\alpha\,|a_0|^2}{\kappa(i\Omega_{\text{p}}-\gamma_{\text{th}})/2 - iG\gamma_{\text{th}}\alpha\,|a_0|^2} \end{aligned} \tag{43}$$

$$\begin{aligned} |r|^2 &= \frac{4G^2\gamma_{\text{th}}^2\alpha^2\,|a_0|^4\,/\kappa^2}{(2G\gamma_{\text{th}}\alpha\,|a_0|^2\,/\kappa-\Omega_{\text{p}})^2+\gamma_{\text{th}}^2} \\ &= \frac{C^2}{(C-\Omega_{\text{p}}/\gamma_{\text{th}})^2+1} \end{aligned} \tag{44}$$

$$C = 2G\alpha\,|a_0|^2\,/\kappa \tag{45}$$

The reflectivity exceeds one in the case of $C > 1$ which can be easily reailzed by enhancing the pump power. The Lorentzian of width is

$$\Gamma_{th} = 2\gamma_{\text{th}} \tag{46}$$

The reflection group delay

$$
\begin{aligned}
r &= \frac{-iC}{i(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C) - 1} \\
&= \frac{-C(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C) + iC}{(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C)^2 + 1} \qquad (47)\\
\phi_r(\Omega_\mathrm{p}) &= \arctan(-\frac{1}{\Omega_\mathrm{p}/\gamma_\mathrm{th} - \mathrm{C}}) \qquad (48)\\
\tau_r &= -\frac{d\phi_r(\Omega_\mathrm{p})}{d\Omega_\mathrm{p}} \\
&= \frac{1}{1 + (\frac{1}{\Omega_\mathrm{p}/\gamma_\mathrm{th} - C})^2} \frac{1/\gamma_\mathrm{th}}{(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C)^2} \\
&= \frac{1/\gamma_\mathrm{th}}{(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C)^2 + 1} \qquad (49)\\
\tau_r &= \mathbf{R}\{\frac{-i}{r} \frac{dr}{d\Omega_\mathrm{p}}\} \qquad (50)
\end{aligned}
$$

The transmission group delay

$$
\begin{aligned}
t &= 1 - \frac{-iC}{i(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C) - 1} \qquad (51)\\
\tau_t &= \mathbf{R}\{\frac{-i}{t} \frac{dt}{d\Omega_\mathrm{p}}\} \\
&= \mathbf{R}\{\frac{C\varepsilon(\gamma_\mathrm{th} + i\Omega_\mathrm{p})(i\varepsilon - \Omega_\mathrm{p} + C\varepsilon)}{(\gamma_\mathrm{th}^2 + \Omega_\mathrm{p}^2)(C^2\gamma_\mathrm{th}^2 - 2C\varepsilon\Omega_\mathrm{p} + \gamma_\mathrm{th}^2 + \Omega_\mathrm{p}^2)}\} \\
&= \mathbf{R}\{\frac{1}{\gamma_\mathrm{th}} \frac{C(1 + i\Omega_\mathrm{p}/\gamma_\mathrm{th})(i - \Omega_\mathrm{p}/\gamma_\mathrm{th} + C)}{[1 + (\Omega_\mathrm{p}/\gamma_\mathrm{th})^2][(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C)^2 + 1]}\} \\
&= \mathbf{R}\{\frac{1}{\gamma_\mathrm{th}} \frac{C(-2\Omega_\mathrm{p}/\gamma_\mathrm{th} + C)}{[1 + (\Omega_\mathrm{p}/\gamma_\mathrm{th})^2][(\Omega_\mathrm{p}/\gamma_\mathrm{th} - C)^2 + 1]}\} \qquad (52)
\end{aligned}
$$