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Supplementary Materials for

The coherence of light is fundamentally tied to the quantum coherence of the emitting particle

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The coherence of light is fundamentally tied to the quantum coherence of the emitting particle: Supplementary Material

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S1. Electric field autocorrelations for spontaneous emission by free electrons

First, we consider a system of a Dirac electron and radiation field with an initial density operator

$$\boldsymbol{\rho}_i = \sum_{\mathbf{i},\mathbf{i}'} \rho_{\mathrm{el}}(\mathbf{i},\mathbf{i}')|\mathbf{i}0\rangle\langle\mathbf{i}'0|\,, \qquad \text{(S1.1)}$$

where $\mathbf{i} = (\mathbf{k}_i, s_i)$ are pure spinor states with wavefunctions [78]

$$\boldsymbol{\psi}_{\mathbf{i}} = \langle\mathbf{r}|\mathbf{i}\rangle = \frac{1}{\sqrt{(2\pi)^3}}\mathbf{u}_{s_i}(\mathbf{k}_i)e^{i\mathbf{k}_i\cdot\mathbf{r}-i\frac{E_i}{\hbar}t}, \qquad \text{(S1.2)}$$

where $E_i = \sqrt{\hbar^2k^2c^2 + m^2c^4}$ is the initial electron energy and $\mathbf{u}_{s_i}(\mathbf{k}_i)$ denotes the Dirac particle spinor. The EM field is quantized in a weakly dispersive, homogeneous medium [79] as

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{q}\sigma}\sqrt{\frac{\hbar v_{g\mathbf{q}\sigma}}{2\omega_{\mathbf{q}\sigma}\epsilon_0 n_{\mathbf{q}\sigma}c}}\boldsymbol{u}_{\mathbf{q}\sigma}(\mathbf{r})e^{-i\omega_{\mathbf{q}\sigma}t}a_{\mathbf{q}\sigma} + h.c.\,, \qquad \text{(S1.3)}$$

with the $\boldsymbol{u}_{\mathbf{q}\sigma}(\mathbf{r})$'s denoting the EM spatial modes. After the interaction, the total density operator to first order in QED (post-selected to include only emission events via the projection operator $P = 1 - \sum_{\mathbf{f}}|\mathbf{f}0\rangle\langle\mathbf{f}0|$) is

$$\boldsymbol{\rho}_f = \sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i},\mathbf{i}')\sum_{\mathbf{f};\mathbf{q}\sigma}M_{\mathbf{fi};\mathbf{q}\sigma}\sum_{\mathbf{f}';\mathbf{q}'\sigma'}M^*_{\mathbf{f}'\mathbf{i}';\mathbf{q}'\sigma'}\left|\mathbf{f}1_{\mathbf{q}\sigma}\right\rangle\left\langle\mathbf{f}'1_{\mathbf{q}'\sigma'}\right|, \qquad \text{(S1.4)}$$

where $M_{\mathbf{fi};\mathbf{q}\sigma} = \frac{i}{\hbar}ec\int d\tau\Theta(t-\tau)\left\langle\mathbf{f};1_{\mathbf{q}\sigma}\middle|\boldsymbol{\alpha}\cdot\mathbf{A}\middle|\mathbf{i};0\right\rangle$ is the transition matrix element, and $\Theta(t)$ denotes the Heaviside step function. Explicitly the transition matrix element is given by

$$M_{\mathbf{fi};\mathbf{q}\sigma} = \frac{i}{\hbar}\sqrt{\frac{\hbar v_{g\mathbf{q}\sigma}}{2\omega_{\mathbf{q}\sigma}\epsilon_0 n_{\mathbf{q}\sigma}c}}\int d^3\mathbf{r}\,\boldsymbol{u}^*_{\mathbf{q}\sigma}(\mathbf{r})\cdot\int d\tau\Theta(t-\tau)\left[ec\boldsymbol{\psi}^\dagger_{\mathbf{f}}(\mathbf{r},\tau)\boldsymbol{\alpha}\boldsymbol{\psi}_{\mathbf{i}}(\mathbf{r},\tau)\right]e^{i\omega_{\mathbf{q}\sigma}\tau}\,, \qquad \text{(S1.5)}$$

The expression in the square brackets can be identified as the matrix element of the 3-current ***operator*** in first quantization:

$$\mathbf{j}(\mathbf{r}) = e\mathbf{n}(\mathbf{r})\mathbf{v} = e\delta(\mathbf{r}-\hat{\mathbf{r}})c\boldsymbol{\alpha}, \qquad \text{(S1.6)}$$

The non-relativistic version is

$$\mathbf{j}(\mathbf{r}) = e\mathbf{n}(\mathbf{r})\mathbf{v} = \frac{e}{m}\delta(\mathbf{r}-\hat{\mathbf{r}})\mathbf{p}, \qquad \text{(S1.7)}$$

Proof:

$$\begin{aligned}\langle\mathbf{f}|\mathbf{j}|\mathbf{i}\rangle = \langle\mathbf{f}(t)|\mathbf{j}(\mathbf{r})|\mathbf{i}(t)\rangle &= ec\langle\mathbf{f}(t)|\delta(\mathbf{r}-\hat{\mathbf{r}})\boldsymbol{\alpha}|\mathbf{i}(t)\rangle \\ &= ec\int d^3\mathbf{r}'\langle\mathbf{f}(t)|\delta(\mathbf{r}-\hat{\mathbf{r}})|\mathbf{r}'\rangle\langle\mathbf{r}'|\boldsymbol{\alpha}|\mathbf{i}(t)\rangle \\ &= ec\int d^3\mathbf{r}'\delta(\mathbf{r}-\mathbf{r}')\langle\mathbf{f}(t)|\mathbf{r}'\rangle\langle\mathbf{r}'|\boldsymbol{\alpha}|\mathbf{i}(t)\rangle = \langle\mathbf{f}(t)|\mathbf{r}\rangle\langle\mathbf{r}|\boldsymbol{\alpha}|\mathbf{i}(t)\rangle \\ &= ec\boldsymbol{\psi}_{\mathbf{f}}^{\dagger}(\mathbf{r},t)\boldsymbol{\alpha}\boldsymbol{\psi}_{\mathbf{i}}(\mathbf{r},t), \qquad \text{(S1.8)}\end{aligned}$$

And thus, writing $\mathbf{j}_{\mathrm{fi}}(\mathbf{r},t) = \langle\mathbf{f}|\mathbf{j}|\mathbf{i}\rangle$

$$M_{\mathbf{fi};\mathbf{q}\sigma} = \frac{i}{\hbar}\sqrt{\frac{\hbar v_{g\mathbf{q}\sigma}}{2\omega_{\mathbf{q}\sigma}\epsilon_0 n_{\mathbf{q}\sigma}c}}\int d^3\mathbf{r}\,\boldsymbol{u}_{\mathbf{q}\sigma}^{*}(\mathbf{r})\cdot\int d\tau\Theta(t-\tau)\mathbf{j}_{\mathrm{fi}}(\mathbf{r},\tau)e^{i\omega_{\mathbf{q}\sigma}\tau}, \qquad \text{(S1.9)}$$

When we detect only the photon component of the state, the electron's degrees of freedom are traced out, and the photon is described by a reduced density operator

$$\boldsymbol{\rho}_{\mathrm{ph}} = \mathrm{Tr}_{\mathrm{el}}\{\boldsymbol{\rho}_f\} = \sum_{\mathbf{q}\sigma}\sum_{\mathbf{q}'\sigma'}\sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')M_{\mathbf{fi};\mathbf{q}\sigma}M_{\mathbf{fi}';\mathbf{q}'\sigma'}^{*}\,|1_{\mathbf{q}\sigma}\rangle\langle 1_{\mathbf{q}'\sigma'}|, \qquad \text{(S1.10)}$$

The positive frequency part of the electromagnetic field operator is

$$\mathbf{E}^{(+)}(\mathbf{r},t) = i\sum_{\mathbf{q}\sigma}\sqrt{\frac{\hbar\omega_{\mathbf{q}\sigma}v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}}\boldsymbol{u}_{\mathbf{q}\sigma}(\mathbf{r})e^{-i\omega_{\mathbf{q}\sigma}t}a_{\mathbf{q}\sigma}, \qquad \text{(S1.11)}$$

We now show that, in the frequency domain, the field-field correlations can be expressed via the current-current correlations of the electrons and the dyadic Green's function [53] of the medium (Eq. S1.19 below):

$$\langle\mathbf{E}^{\dagger}(\mathbf{r}',\omega')\mathbf{E}(\mathbf{r},\omega)\rangle = \mu_0^2\omega\omega'\int d^3\mathbf{R}'\,\mathbf{G}^{\dagger}(\mathbf{r}',\mathbf{R}',\omega')\int d^3\mathbf{R}\,\mathbf{G}(\mathbf{r},\mathbf{R},\omega)\langle\mathbf{j}^{\dagger}(\mathbf{R}',\omega')\mathbf{j}(\mathbf{R},\omega)\rangle_{\mathrm{el}}$$

Derivation:

According to Eqs. S1.10-11, the time-domain field-field correlations are

$$\langle\mathbf{E}^{(-)}(\mathbf{r}',t')\mathbf{E}^{(+)}(\mathbf{r},t)\rangle = \mathrm{Tr}\{\boldsymbol{\rho}_{\mathrm{ph}}\mathbf{E}^{(-)}(\mathbf{r}',t')\mathbf{E}^{(+)}(\mathbf{r},t)\}$$

$$= \sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')\sum_{\mathbf{q}\sigma}\sum_{\mathbf{q}'\sigma'}M_{\mathbf{fi};\mathbf{q}\sigma}(t)M_{\mathbf{fi}';\mathbf{q}'\sigma'}^{*}(t')\,\langle 1_{\mathbf{q}'\sigma'}|\mathbf{E}^{(-)}(\mathbf{r}',t')\mathbf{E}^{(+)}(\mathbf{r},t)|1_{\mathbf{q}\sigma}\rangle$$

$$= \sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')\sum_{\mathbf{q}\sigma}\sum_{\mathbf{q}'\sigma'}M_{\mathbf{fi};\mathbf{q}\sigma}(t)M_{\mathbf{fi}';\mathbf{q}'\sigma'}^{*}(t')\sqrt{\frac{\hbar\omega_{\mathbf{q}\sigma}v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}}\sqrt{\frac{\hbar\omega_{\mathbf{q}'\sigma'}v_{g\mathbf{q}'\sigma'}}{2\epsilon_0 n_{\mathbf{q}'\sigma'}c}}\boldsymbol{u}_{\mathbf{q}'\sigma'}^{*}(\mathbf{r}')e^{i\omega_{\mathbf{q}'\sigma'}t'}\boldsymbol{u}_{\mathbf{q}\sigma}(\mathbf{r})e^{-i\omega_{\mathbf{q}\sigma}t},$$

(S1.12)

Substituting the expressions for $M_{\mathbf{fi};\mathbf{q}\sigma}(t)$ and employing index notation:

$$\left\langle E_\alpha^{(-)}(\mathbf{r}',t')E_\beta^{(+)}(\mathbf{r},t)\right\rangle = \sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}') \times \int d^3\mathbf{R}' \int d\tau' \left[\Theta(t'-\tau')\sum_{\mathbf{q}'\sigma'}\frac{v_{g\mathbf{q}'\sigma'}}{2\epsilon_0 n_{\mathbf{q}'\sigma'}c}\mathcal{u}^*_{\mathbf{q}'\sigma',\alpha}(\mathbf{r}')\mathcal{u}_{\mathbf{q}'\sigma',\gamma}(\mathbf{R}')e^{i\omega_{\mathbf{q}'\sigma'}(t'-\tau')}\right] j^*_{\mathrm{fi}',\gamma}(\mathbf{R}',\tau') \times \int d^3\mathbf{R}\int d\tau\left[\Theta(t-\tau)\sum_{\mathbf{q}\sigma}\frac{v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})e^{-i\omega_{\mathbf{q}\sigma}(t-\tau)}\right] j_{\mathrm{fi},\delta}(\mathbf{R},\tau), \quad \text{(S1.13)}$$

Note that the Fourier transform of the square brackets is

$$\int dt e^{i\omega t}\,\Theta(t)\sum_{\mathbf{q}\sigma}\frac{v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})e^{-i(\omega_{\mathbf{q}\sigma}-i0^+)t} = i\sum_{\mathbf{q}\sigma}\frac{v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}\frac{\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})}{\omega-\omega_{\mathbf{q}\sigma}+i0^+}, \quad \text{(S1.14)}$$

and that the dyadic Green tensor [53] is

$$G_{\beta\delta}(\mathbf{r},\mathbf{R},\omega) = \sum_{\mathbf{q}\sigma}\frac{cv_{g\mathbf{q}\sigma}}{n_{\mathbf{q}\sigma}}\frac{\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})}{\omega^2_{\mathbf{q}\sigma}-\omega^2} = -\sum_{\mathbf{q}\sigma}\frac{cv_{g\mathbf{q}\sigma}}{2\omega_{\mathbf{q}\sigma}n_{\mathbf{q}\sigma}}\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})\left[\frac{1}{\omega-\omega_{\mathbf{q}\sigma}+i0^+}-\frac{1}{\omega+\omega_{\mathbf{q}\sigma}+i0^+}\right] = -\frac{1}{\mu_0\omega}\sum_{\mathbf{q}\sigma}\frac{v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}\frac{\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})}{\omega-\omega_{\mathbf{q}\sigma}+i0^+} - \frac{1}{\mu_0\omega}\sum_{\mathbf{q}\sigma}\frac{v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}\frac{\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})}{\omega+\omega_{\mathbf{q}\sigma}+i0^+}, \quad \text{(S1.15)}$$

Therefore Eq. (S1.14) can be simplified to

$$i\sum_{\mathbf{q}\sigma}\frac{v_{g\mathbf{q}\sigma}}{2\epsilon_0 n_{\mathbf{q}\sigma}c}\frac{\mathcal{u}_{\mathbf{q}\sigma,\beta}(\mathbf{r})\mathcal{u}^*_{\mathbf{q}\sigma,\delta}(\mathbf{R})}{\omega-\omega_{\mathbf{q}\sigma}+i0^+} = -i\Theta(\omega)\mu_0\omega G_{\beta\delta}(\mathbf{r},\mathbf{R},\omega), \quad \text{(S1.16)}$$

i.e., the positive frequency part of the Green tensor. Now define the frequency field in the following manner:

$$\mathbf{E}^{(+)}(\mathbf{r},t) = \int_0^\infty d\omega e^{-i\omega t}\mathbf{E}(\mathbf{r},\omega)$$

$$\mathbf{E}^{(-)}(\mathbf{r},t) = \int_0^\infty d\omega e^{i\omega t}\mathbf{E}^\dagger(\mathbf{r},\omega)$$

Identifying Eq. (S1.13) as a convolution and moving to the frequency domain one has

$$\left\langle E_\alpha^\dagger(\mathbf{r}',\omega')E_\beta(\mathbf{r},\omega)\right\rangle = \mu_0^2\omega\omega'\int d^3\mathbf{R}'\,G^*_{\alpha\gamma}(\mathbf{r}',\mathbf{R}',\omega')\int d^3\mathbf{R}\,G_{\beta\delta}(\mathbf{r},\mathbf{R},\omega) \times\sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')j^*_{\mathrm{fi}',\gamma}(\mathbf{R}',\omega')j_{\mathrm{fi},\delta}(\mathbf{R},\omega), \quad \text{(S1.19)}$$

The double-sum expression in the above equation can be simplified further, because

$$\sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')\mathbf{j}^{*}_{\mathrm{fi}'}(\mathbf{R}',\omega')\mathbf{j}_{\mathrm{fi}}(\mathbf{R},\omega)=\sum_{\mathbf{f}}\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')\langle\mathbf{i}'|\mathbf{j}^{\dagger}(\mathbf{R}',\omega')|\mathbf{f}\rangle\langle\mathbf{f}|\mathbf{j}(\mathbf{R},\omega)|\mathbf{i}\rangle$$
$$=\sum_{\mathbf{i},\mathbf{i}'}\rho_{\mathrm{el}}(\mathbf{i};\mathbf{i}')\langle\mathbf{i}'|\mathbf{j}^{\dagger}(\mathbf{R}',\omega')\mathbf{j}(\mathbf{R},\omega)|\mathbf{i}\rangle=\mathrm{Tr}\{\rho_{\mathrm{el}}\mathbf{j}^{\dagger}(\mathbf{R}',\omega')\mathbf{j}(\mathbf{R},\omega)\}$$
$$=\langle\mathbf{j}^{\dagger}(\mathbf{R}',\omega')\mathbf{j}(\mathbf{R},\omega)\rangle_{\mathrm{el}}$$

Hence

$$\langle\mathbf{E}^{\dagger}(\mathbf{r}',\omega')\mathbf{E}(\mathbf{r},\omega)\rangle=\mu_0^2\omega\omega'\int d^3\mathbf{R}'\,\mathbf{G}^{\dagger}(\mathbf{r}',\mathbf{R}',\omega')\int d^3\mathbf{R}\,\mathbf{G}(\mathbf{r},\mathbf{R},\omega)\langle\mathbf{j}^{\dagger}(\mathbf{R}',\omega')\mathbf{j}(\mathbf{R},\omega)\rangle_{\mathrm{el}},\quad \text{(S1.19)}$$

As we are considering only positive frequencies in the first place when calculating $\left\langle E_{\alpha}^{(-)}(\mathbf{r}',t')E_{\beta}^{(+)}(\mathbf{r},t)\right\rangle$, we might as well use the full expression $G_{\beta\delta}(\mathbf{r},\mathbf{R},t)$.

Thus, substituting the Green tensor we find

$$\langle\mathbf{E}^{(-)}(\mathbf{r}',t')\mathbf{E}^{(+)}(\mathbf{r},t)\rangle=\mu_0^2\frac{\partial}{\partial t'}\frac{\partial}{\partial t}\int d^3\mathbf{R}'\int d\tau'\mathbf{G}^{\dagger}(\mathbf{r}',\mathbf{R}',t'-\tau')\int d^3\mathbf{R}\int d\tau\mathbf{G}(\mathbf{r},\mathbf{R},t-\tau)\times\langle\mathbf{j}(\mathbf{R}',\tau')\mathbf{j}(\mathbf{R},\tau)\rangle_{\mathrm{el}},\quad \text{(S1.20)}$$

So, a knowledge of the current correlations $\langle\mathbf{j}^{\dagger}(\mathbf{R}',\omega')\mathbf{j}(\mathbf{R},\omega)\rangle_{\mathrm{el}}$ of the electrons provides the value for the electric field autocorrelation $\langle\mathbf{E}^{\dagger}(\mathbf{r}',\omega')\mathbf{E}(\mathbf{r},\omega)\rangle$.

S2. Current correlations of free quantum emitters

By a "free quantum emitter", we refer to particles which emit waves as they propagate through a medium due to recoil of their center of mass motion. Below we detail how to derive the current correlations for an arbitrary number of particles, wherein electrons emitting Cherenkov radiation are considered as a case study.

First quantization

In first quantization, a generalization of the current operator to N particles is:

$$\mathbf{j}(\mathbf{r})=\frac{e}{m}\sum_{i=1}^{N}\delta(\mathbf{r}^{(i)}-\mathbf{r})\mathbf{p}^{(i)},\qquad \text{(S2.1)}$$

in the non-relativistic case, and

$$\mathbf{j}(\mathbf{r})=ec\sum_{i=1}^{N}\delta(\mathbf{r}^{(i)}-\mathbf{r})\boldsymbol{\alpha}^{(i)},\qquad \text{(S2.2)}$$

in the relativistic case. In order to avoid the redundancy of first quantization when dealing with identical particles, from now on we'll employ second quantization to describe both the current operator and the electron state ρ_{el}.

Second quantization: nonrelativistic current

In second quantization of either bosons, $\left[a_{\mathbf{k}}, a_{\mathbf{k}'}^{\dagger}\right] = \delta_{\mathbf{k}\mathbf{k}'}$ or fermions $\left\{a_{\mathbf{k}}, a_{\mathbf{k}'}^{\dagger}\right\} = \delta_{\mathbf{k}\mathbf{k}'}$ the current density *operator* of free, massive particles described by nonrelativistic quantum mechanics is

$$\mathbf{j}(\mathbf{r},t) = \frac{e\hbar}{2im}\left[\psi^{\dagger}(\mathbf{r},t)\boldsymbol{\nabla}\psi(\mathbf{r},t) - \left(\boldsymbol{\nabla}\psi^{\dagger}(\mathbf{r},t)\right)\psi(\mathbf{r},t)\right], \qquad \text{(S2.3)}$$

where $\psi(\mathbf{r},t) = \sum_{\mathbf{k}} \frac{e^{i\left(\mathbf{k}\cdot\mathbf{r} - \frac{E_{\mathbf{k}}}{\hbar}t\right)}}{\sqrt{V}} a_{\mathbf{k}}$ is the position space annihilation operator in the Heisenberg picture for a free particle. First let's make a few approximation steps. Substituting into the current operator we find

$$\mathbf{j}(\mathbf{r},t) = \frac{e\hbar}{2mV}\left[\sum_{\mathbf{k}'}\sum_{\mathbf{k}} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} e^{-i\frac{E_{\mathbf{k}}-E_{\mathbf{k}'}}{\hbar}t}(\mathbf{k}+\mathbf{k}')\, a_{\mathbf{k}'}^{\dagger} a_{\mathbf{k}}\right], \qquad \text{(S2.4)}$$

Taking the Fourier transform:

$$\mathbf{j}(\mathbf{q},t) = \int d^3\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\mathbf{j}(\mathbf{r},t) = \frac{e\hbar}{mV}\sum_{\mathbf{k}} e^{i\frac{E_{\mathbf{k}-\mathbf{q}}-E_{\mathbf{k}}}{\hbar}t}\left(\mathbf{k} - \frac{\mathbf{q}}{2}\right) a_{\mathbf{k}-\mathbf{q}}^{\dagger} a_{\mathbf{k}}, \qquad \text{(S2.5)}$$

In the paraxial and zero-recoil limits, we find for both bosons and fermions

$$\mathbf{j}(\mathbf{q},t) = e\mathbf{v}_0 e^{-i\mathbf{q}\cdot\mathbf{v}_0 t}\frac{1}{V}\sum_{\mathbf{k}} a_{\mathbf{k}-\mathbf{q}}^{\dagger} a_{\mathbf{k}}, \qquad \text{(S2.6)}$$

where $\mathbf{v}_0$ denotes the emitter's carrier velocity. Going back to position representation:

$$\mathbf{j}(\mathbf{r},t) = e\mathbf{v}_0\psi^{\dagger}(\mathbf{r}-\mathbf{v}_0 t)\psi(\mathbf{r}-\mathbf{v}_0 t), \qquad \text{(S2.7)}$$

Second quantization: relativistic current

Here, we start from the 4-current

$$j^{\mu} = \bar{\psi}\gamma^{\mu}\psi = \psi^{\dagger}\gamma^{0}\gamma^{\mu}\psi, \qquad \text{(S2.8)}$$

The 3-current is, in turn,

$$j^{i} = \psi^{\dagger}\gamma^{0}\gamma^{i}\psi = \psi^{\dagger}\alpha^{i}\psi, \qquad \text{(S2.9)}$$

We now derive the relativistic analogue of Eq. (S2.7):

Derivation:
Let us write

$$\psi_{\sigma}(\mathbf{r},t) = \sum_{\mathbf{p},\sigma} a_{\mathbf{p}\sigma}\mathbf{u}_{\mathbf{p}\sigma}\frac{e^{i(\mathbf{p}\cdot\mathbf{r} - E_{\mathbf{p}\sigma}t)/\hbar}}{\sqrt{V}}, \qquad \text{(S2.10)}$$

with $\sigma = \uparrow_+, \downarrow_+, \uparrow_-, \downarrow_-$ denoting spin and particle/antiparticle. Substituting, we have that

$$\mathbf{j}(\mathbf{r},t) = ec\psi^{\dagger}\boldsymbol{\alpha}\psi = \frac{ec}{V}\sum_{\mathbf{p},\sigma}\sum_{\mathbf{p}',\sigma'} a_{\mathbf{p}'\sigma'}^{\dagger} a_{\mathbf{p}\sigma}\mathbf{u}_{\mathbf{p}'\sigma'}^{\dagger}\boldsymbol{\alpha}\mathbf{u}_{\mathbf{p}\sigma} e^{i\left[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r} - \left(E_{\mathbf{p}\sigma} - E_{\mathbf{p}'\sigma'}\right)t\right]/\hbar}, \qquad \text{(S2.11)}$$

Although a thorough treatment of the role of spin and recoil corrections in CR was discussed in Ref. [44], for simplicity we choose here to neglect any recoil and spin-flip contributions to the current operator. Recoil corrections are typically orders of magnitude smaller than the electron momentum, making this approximation quite accurate. Also, we keep only contributions where a particle scatters to a particle (no particle-antiparticle transitions), and thus restrict ourselves to $\sigma =\uparrow,\downarrow$. Again, employing the zero-recoil and paraxial approximations, we find that:

$$\mathbf{u}^{\dagger}_{\mathbf{p}'\sigma'}\boldsymbol{\alpha}\mathbf{u}_{\mathbf{p}\sigma} \cong \frac{1}{c}\mathbf{v}_0\delta_{\sigma\sigma'}, \qquad \text{(S2.12)}$$

$$E_{\mathbf{p}} - E_{\mathbf{p}'} \cong (\mathbf{p}-\mathbf{p}')\cdot\mathbf{v}_0, \qquad \text{(S2.13)}$$

$$\mathbf{j}(\mathbf{r},t) \cong \frac{e\mathbf{v}_0}{V}\sum_{\sigma}\sum_{\mathbf{k},\mathbf{k}'} a^{\dagger}_{\mathbf{k}'\sigma}a_{\mathbf{k}\sigma}e^{i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}-\mathbf{v}_0t)}, \qquad \text{(S2.14)}$$

where now the carrier velocity is $\mathbf{v}_0 = \frac{\mathbf{p}_0}{m\gamma}$. Now, since $\{a_{\mathbf{k}\sigma}, a^{\dagger}_{\mathbf{k}'\sigma'}\} = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\sigma\sigma'}$, if we redefine

$$\psi_\sigma(\mathbf{r}-\mathbf{v}_0t) = \sum_{\mathbf{k}} a_{\mathbf{k}\sigma}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{v}_0t)}}{\sqrt{V}}, \qquad \text{(S2.15)}$$

Then we can keep the correct spatial anti-commutation

$$\{\psi_\sigma(\mathbf{r}-\mathbf{v}_0t), \psi^{\dagger}_{\sigma'}(\mathbf{r}'-\mathbf{v}_0t)\} = \sum_{\mathbf{k}}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{v}_0t)}}{\sqrt{V}}\sum_{\mathbf{k}'}\{a_{\mathbf{k}\sigma}, a^{\dagger}_{\mathbf{k}'\sigma'}\}\frac{e^{-i\mathbf{k}'\cdot(\mathbf{r}'-\mathbf{v}_0t)}}{\sqrt{V}}$$

$$= \delta_{\sigma\sigma'}\sum_{\mathbf{k}}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{V} = \delta_{\sigma\sigma'}\delta(\mathbf{r}-\mathbf{r}'), \qquad \text{(S2.16)}$$

From the above derivation, it follows that

$$\mathbf{j}(\mathbf{r},t) \cong e\mathbf{v}_0\sum_{\sigma}\psi^{\dagger}_{\sigma}(\mathbf{r}-\mathbf{v}_0t)\psi_\sigma(\mathbf{r}-\mathbf{v}_0t), \qquad \text{(S2.17)}$$

In close analogy to Eq. (S2.7), with the changes being that $\mathbf{v}_0 = \frac{\mathbf{p}_0}{m\gamma}$ is now the relativistic expression for the velocity, and the introduction of the spin degree of freedom for the electrons.

Current correlations

Since, for both nonrelativistic and relativistic particles, the current operator obtains a similar form, we may proceed by analysing the general behaviour. The current correlations appearing in Eq. (S1.20) can be rewritten

$$\langle j_\alpha(\mathbf{r}',t')j_\beta(\mathbf{r},t)\rangle_{\mathrm{el}} = e^2 v_{0\alpha}\, v_{0\beta}\sum_{\sigma'}\sum_{\sigma}\langle\psi^{\dagger}_{\sigma'}(\mathbf{r}'-\mathbf{v}_0t')\psi_{\sigma'}(\mathbf{r}'-\mathbf{v}_0t')\psi^{\dagger}_{\sigma}(\mathbf{r}-\mathbf{v}_0t)\psi_\sigma(\mathbf{r}-\mathbf{v}_0t)\rangle, \qquad \text{(S2.18)}$$

We bring the expression $\langle\psi^{\dagger}_{\sigma'}\psi_{\sigma'}\psi^{\dagger}_{\sigma}\psi_\sigma\rangle$ to normal ordering using the commutation relations for bosons, or anticommutation relations for fermions, to obtain the same results:

$$\begin{aligned}\langle j_\alpha(\mathbf{r}',t')j_\beta(\mathbf{r},t)\rangle_{\mathrm{el}} &= e^2 v_{0\alpha}\, v_{0\beta}\sum_{\sigma'}\sum_{\sigma}\langle\psi^{\dagger}_{\sigma'}(\mathbf{r}'-\mathbf{v}_0t')\psi^{\dagger}_{\sigma}(\mathbf{r}-\mathbf{v}_0t)\psi_\sigma(\mathbf{r}-\mathbf{v}_0t)\psi_{\sigma'}(\mathbf{r}'-\mathbf{v}_0t')\rangle \\ &+ e^2 v_{0\alpha}\, v_{0\beta}\delta[\mathbf{r}-\mathbf{r}'-\mathbf{v}_0(t-t')]\sum_{\sigma}\langle\psi^{\dagger}_{\sigma}(\mathbf{r}-\mathbf{v}_0t)\psi_\sigma(\mathbf{r}-\mathbf{v}_0t)\rangle, \qquad \text{(S2.19)}\end{aligned}$$

Now, using the first and second order correlation functions of the emitters as

$$G^{(1)}_{\mathrm{e}}(\mathbf{x},\mathbf{x}) = \sum_{\sigma}\langle\psi^{\dagger}_{\sigma}(\mathbf{x})\psi_\sigma(\mathbf{x})\rangle = \rho_{\mathrm{e}}(\mathbf{x},\mathbf{x}), \qquad \text{(S2.20)}$$

$$G^{(2)}_{\mathrm{e}}(\mathbf{x}',\mathbf{x}) = \sum_{\sigma'}\sum_{\sigma}\langle\psi^{\dagger}_{\sigma'}(\mathbf{x}')\psi^{\dagger}_{\sigma}(\mathbf{x})\psi_\sigma(\mathbf{x})\psi_{\sigma'}(\mathbf{x}')\rangle, \qquad \text{(S2.21)}$$

with $\mathbf{x} = \mathbf{r} - \mathbf{v}_0 t$ and $\rho_\mathrm{e}(\mathbf{x},\mathbf{x})$ is the diagonal of the emitter density matrix, so that

$$\left\langle j_\alpha(\mathbf{x}') j_\beta(\mathbf{x}) \right\rangle_\mathrm{el} = e^2 v_{0\alpha} v_{0\beta} G^{(2)}(\mathbf{x}',\mathbf{x}) + e^2 v_{0\alpha} v_{0\beta} \delta(\mathbf{x}-\mathbf{x}') G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x}), \qquad \text{(S2.22)}$$

And in Fourier space

$$\begin{aligned}\left\langle j_\alpha^\dagger(\mathbf{q}',\omega') j_\beta(\mathbf{q},\omega) \right\rangle_\mathrm{el} \\ &= \int dt' \int d^3\mathbf{r}'\, e^{-i(\mathbf{q}'\cdot\mathbf{r}'-\omega' t')} \int dt \int d^3\mathbf{r} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} \left\langle j_\alpha^\dagger(\mathbf{r}',t') j_\beta(\mathbf{r},t) \right\rangle_\mathrm{el} \\ &= (2\pi)^2 e^2 v_{0\alpha} v_{0\beta} \delta(\omega' - \mathbf{q}'\cdot\mathbf{v}_0) \delta(\omega - \mathbf{q}\cdot\mathbf{v}_0) \\ &\times \left[\iint d^3\mathbf{x} \int d^3\mathbf{x}'\, e^{i\mathbf{q}\cdot\mathbf{x} - i\mathbf{q}'\cdot\mathbf{x}'} G_\mathrm{e}^{(2)}(\mathbf{x}',\mathbf{x}) + \int d^3\mathbf{x} e^{i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{x}} G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x}) \right],\end{aligned}$$

(S2.23)

S3. Field correlations of Cherenkov radiation and the uncertainty principle

For Cherenkov radiation the far field Green function [53] is

$$\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) = \frac{e^{iqr}}{4\pi r} (\mathbf{I} - \hat{\mathbf{r}}\hat{\mathbf{r}}) e^{-i\mathbf{q}\cdot\mathbf{r}'}, \qquad \text{(S3.1)}$$

where $\mathbf{q} = \hat{\mathbf{r}} n\omega/c$ is the wavevector of the emitted radiation in the observation direction. So, we find

$$\left\langle \mathbf{E}^\dagger(\mathbf{r}',\omega') \mathbf{E}(\mathbf{r},\omega) \right\rangle = e^2 \mu_0^2 \omega\omega' \frac{e^{-iq'r'}}{4\pi r'} \frac{e^{iqr}}{4\pi r} (\mathbf{I} - \hat{\mathbf{r}}'\hat{\mathbf{r}}')(\mathbf{I} - \hat{\mathbf{r}}\hat{\mathbf{r}}) \left\langle \mathbf{j}^\dagger(\mathbf{q}',\omega') \mathbf{j}(\mathbf{q},\omega) \right\rangle_\mathrm{e},$$

(S3.2)

Substituting the emitter current correlations we have

$$\begin{aligned}\left\langle \mathbf{E}^\dagger(\mathbf{r}',\omega') \mathbf{E}(\mathbf{r},\omega) \right\rangle \\ &= e^2 \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}' \sin\theta \sin\theta'\, \mu_0^2 \omega\omega' v_0^2 \frac{e^{-iq'r'}}{4\pi r'} \frac{e^{iqr}}{4\pi r} \\ &\times (2\pi)^2 \delta(\omega' - \mathbf{q}'\cdot\mathbf{v}_0) \delta(\omega - \mathbf{q}\cdot\mathbf{v}_0) \\ &\times \left[\iint d^3\mathbf{x} \int d^3\mathbf{x}'\, e^{i\mathbf{q}\cdot\mathbf{x} - i\mathbf{q}'\cdot\mathbf{x}'} G_\mathrm{e}^{(2)}(\mathbf{x}',\mathbf{x}) + \int d^3\mathbf{x} e^{i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{x}} G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x}) \right],\end{aligned}$$

(S3.3)

The Cherenkov radiation degree of coherence is simplified by projecting the fields onto the polarization direction $\hat{\boldsymbol{\theta}}$, integrating over the observation direction $\hat{\mathbf{r}}$ (giving the direction of the Cherenkov cone $\hat{\mathbf{r}}_\mathrm{c}$), considering a specific distance $r = r'$ from the emitter and neglecting the medium dispersion (the latter assumption can be relaxed). Also, the radiation is assumed to be detected by a finite bandwidth detector with field transmission $T(\omega)$, centered at ω_0 with a bandwidth of $\Delta\omega_\mathrm{det}$. This gives a scalar relation depending on ω, ω':

$$\begin{aligned}\left\langle E^{(-)}(\omega') E^{(+)}(\omega) \right\rangle \\ &= \frac{\hbar\omega_0 \alpha\beta}{2n\epsilon_0 c r^2} \sin^2\theta_\mathrm{c}\, e^{i\frac{n}{c}(\omega-\omega')r} t_\mathrm{int} T(\omega) T(\omega') \\ &\times \left[\iint d^3\mathbf{x} \int d^3\mathbf{x}'\, e^{i\mathbf{q}_\mathrm{c}\cdot\mathbf{x} - i\mathbf{q}_\mathrm{c}'\cdot\mathbf{x}'} G_\mathrm{e}^{(2)}(\mathbf{x}',\mathbf{x}) + \int d^3\mathbf{x} e^{i(\mathbf{q}_\mathrm{c}-\mathbf{q}_\mathrm{c}')\cdot\mathbf{x}} \rho_\mathrm{e}(\mathbf{x},\mathbf{x}) \right],\end{aligned}$$

(S3.4)

with $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$, $\cos\theta_\mathrm{c} = \frac{1}{n\beta}$ and $\mathbf{q}_{c\omega} = q_\omega \hat{\mathbf{r}}_\mathrm{c}$. Note that by using the delta function properties we have

$$\delta(\omega' - q_{\omega'}\hat{\mathbf{r}}' \cdot \mathbf{v}_0)\delta(\omega - q_\omega \hat{\mathbf{r}} \cdot \mathbf{v}_0) = \frac{2}{n\beta(\omega' + \omega)}\delta(\cos\theta - \cos\theta_\mathrm{c})\frac{t_\mathrm{int}}{2\pi}, \qquad \text{(S3.5)}$$

with $\cos\theta_\mathrm{c} = \frac{\omega + \omega'}{n(\omega)\omega\beta + n(\omega')\omega'\beta} \cong \frac{1}{n\beta}$ and where $\delta(0) = \frac{1}{2\pi}\lim_{t_\mathrm{int}\to\infty}\int_{-t_\mathrm{int}/2}^{t_\mathrm{int}/2} e^{i0t}dt = \frac{t_\mathrm{int}}{2\pi}$.

The detailed derivation of (S3.5):

Note that the following identity holds

$$\delta[f(x)]\delta[g(x)] = 2\delta[f(x) + g(x)]\delta[f(x) - g(x)], \qquad \text{(S3.6)}$$

Proof:

$$\delta[f(x)]\delta[g(x)] = \lim_{\sigma\to 0}\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{f^2(x)}{2\sigma^2}\right]\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{g^2(x)}{2\sigma^2}\right]$$
$$= \lim_{\sigma\to 0}\frac{1}{\sqrt{2\pi}\sigma}\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{f^2(x) + g^2(x)}{2\sigma^2}\right]$$
$$= \lim_{\sigma\to 0}\frac{1}{\sqrt{2\pi}\sigma}\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{[f(x) + g(x)]^2 + [f(x) - g(x)]^2}{4\sigma^2}\right\}$$
$$= 2\lim_{\sigma\to 0}\frac{1}{\sqrt{2\pi}\sqrt{2}\sigma}\exp\left\{-\frac{[f(x) + g(x)]^2}{2\left(\sqrt{2}\sigma\right)^2}\right\}\frac{1}{\sqrt{2\pi}\sqrt{2}\sigma}\exp\left\{-\frac{[f(x) - g(x)]^2}{2\left(\sqrt{2}\sigma\right)^2}\right\}$$
$$= 2\delta[f(x) + g(x)]\delta[f(x) - g(x)], \qquad \text{(S3.7)}$$

So that for our case

$$\delta[\omega - \beta c q(\omega)\cos\theta]\delta[\omega' - \beta c q(\omega')\cos\theta]$$
$$= \frac{2}{q(\omega)c\beta + q(\omega')c\beta}\delta\left[\cos\theta - \frac{\omega + \omega'}{n(\omega)\omega\beta + n(\omega')\omega'\beta}\right]$$
$$\times\,\delta\left[\omega - \omega' - (\omega + \omega')\frac{n(\omega)\omega - n(\omega')\omega'}{n(\omega)\omega + n(\omega')\omega'}\right]$$
$$= \frac{2}{n\beta(\omega' + \omega)}\delta(\cos\theta - \cos\theta_\mathrm{c})$$
$$\times\,\frac{t_\mathrm{int}}{2\pi}\operatorname{sinc}\left\{\frac{t_\mathrm{int}}{2}\left[\omega - \omega' - (\omega + \omega')\frac{n(\omega)\omega - n(\omega')\omega'}{n(\omega)\omega + n(\omega')\omega'}\right]\right\}, \qquad \text{(S3.8)}$$

And, noting that for small frequency intervals $\omega - \omega'$,

$$\omega - \omega' - (\omega + \omega')\frac{n(\omega)\omega - n(\omega')\omega'}{n(\omega)\omega + n(\omega')\omega'} \cong \left(1 - \frac{n_g}{n}\right)(\omega - \omega'), \qquad \text{(S3.9)}$$

thus

$$\delta[\omega - \beta c q(\omega)\cos\theta]\delta[\omega' - \beta c q(\omega')\cos\theta]$$
$$= \frac{2}{n\beta(\omega' + \omega)}\delta(\cos\theta - \cos\theta_c)\frac{t_\mathrm{int}}{2\pi}\operatorname{sinc}\left[\frac{t_\mathrm{int}}{2}\left(1 - \frac{n_g}{n}\right)(\omega - \omega')\right], \qquad \text{(S3.10)}$$

And, for weak dispersion we have $\operatorname{sinc}\left[\frac{t_\mathrm{int}}{2}\left(1 - \frac{n_g}{n}\right)(\omega - \omega')\right] \cong 1$ and $\frac{2}{n\beta(\omega' + \omega)} \cong \frac{1}{n\beta\omega}$.

Taking the temporal Fourier transform (measuring time with respect to the center of the pulse) we find:

$$\left\langle E^{(-)}(t')E^{(+)}(t)\right\rangle = G_\mathrm{rad}^{(1)}(t, t'), \qquad \text{(S3.10)}$$

Where, up to constants we identify the first-order correlation function of the radiation as

$$\left\langle E^{(-)}(t')E^{(+)}(t)\right\rangle$$
$$= A\int d\omega e^{-i\omega t}\int d\omega' e^{i\omega' t'} \underbrace{t_{\text{int}}T(\omega)T(\omega')}_{\text{detection efficiency}}$$
$$\times\left[\underbrace{\int d^3\mathbf{x}\int d^3\mathbf{x}'\, e^{i\mathbf{q}_\mathrm{c}\cdot\mathbf{x}-i\mathbf{q}'_\mathrm{c}\cdot\mathbf{x}'}G_\mathrm{e}^{(2)}(\mathbf{x}',\mathbf{x})+\int d^3\mathbf{x}e^{i(\mathbf{q}_\mathrm{c}-\mathbf{q}'_\mathrm{c})\cdot\mathbf{x}}G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x})}_{\text{emitter statistics}}\right],$$
(S3.11)

where $A=\frac{\hbar\omega_0}{2n\epsilon_0 cr^2}\alpha\beta\sin^2\theta_\mathrm{c}$ is a constant depending on the geometry and strength of the interaction.

For a single particle, let us find the variance of the emitted pulse by taking $t=t'$, and set $G_\mathrm{e}^{(2)}(\mathbf{x}',\mathbf{x})=0$, the observation direction as $\hat{\mathbf{r}}_\mathrm{c}$, and assume weak dispersion with the shockwave travelling at a group velocity v_g. Taking $T(\omega)=\exp\left[-\frac{(\omega-\omega_0)^2}{4\Delta\omega_{\text{det}}^2}\right]$ for the detection efficiency, and noting that

$$T(\omega)T(\omega')=\exp\left[-\frac{(\omega-\omega_0)^2}{4\Delta\omega_{\text{det}}^2}\right]\exp\left[-\frac{(\omega'-\omega_0)^2}{4\Delta\omega_{\text{det}}^2}\right]=\exp\left[-\frac{\delta\omega^2}{8\Delta\omega_{\text{det}}^2}\right]\exp\left[-\frac{(\varpi-\omega_0)^2}{2\Delta\omega_{\text{det}}^2}\right]$$

with $\delta\omega=\omega-\omega'$ and $\varpi=\frac{\omega+\omega'}{2}$, we find for the shockwave power envelope $P_{\text{shw}}(t)$:

$$P_{\text{shw}}(t)\propto\left\langle E^{(-)}(t)E^{(+)}(t)\right\rangle\propto\int d\delta\omega e^{-i\delta\omega t}\exp\left[-\frac{\delta\omega^2}{8\Delta\omega_{\text{det}}^2}\right]\int d^3\mathbf{x}e^{i\delta\omega\frac{\hat{\mathbf{r}}_\mathbf{c}\cdot\mathbf{x}}{v_g}}G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x}),$$
(S3.12)

and, via the use of the convolution theorem:

$$P_{\text{shw}}(t)=\int d\tau\frac{1}{\sqrt{2\pi}\Delta t_{\text{det}}}e^{-\frac{(t-\tau)^2}{2\Delta t_{\text{det}}^2}}\int d^3\mathbf{x}\delta\left(\tau-\frac{\hat{\mathbf{r}}_\mathbf{c}\cdot\mathbf{x}}{v_g}\right)G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x}),\qquad\text{(S3.13)}$$

where we defined the temporal resolution of the detection as $\Delta t_{\text{det}}=\frac{1}{2\Delta\omega_{\text{det}}}$

The temporal variance of $P_{\text{shw}}(t)$ is then simply

$$\Delta t_{\text{shw}}^2=\Delta t_{\text{det}}^2+\frac{\Delta x_\mathrm{e}^2}{v_g^2},\qquad\text{(S3.14)}$$

where

$$\Delta x_\mathrm{e}^2=\int d^3\mathbf{x}(\hat{\mathbf{r}}_\mathbf{c}\cdot\mathbf{x})^2G_\mathrm{e}^{(1)}(\mathbf{x},\mathbf{x}),\qquad\text{(S3.15)}$$

is the variance in the emitter wavefunction size along the $\hat{\mathbf{r}}_\mathrm{c}$ direction. Defining $\Delta x_{\text{shw}}=v_g\Delta t_{\text{shw}}$ we have

$$\Delta x_{\text{shw}}^2=\Delta x_\mathrm{e}^2+v_g^2\Delta t_{\text{det}}^2,\qquad\text{(S3.16)}$$

Thus, for a perfect detection system wherein $\Delta t_{\text{det}}\to 0$ we have $\Delta x_{\text{shw}}=\Delta x_\mathrm{e}$. Applying the Heisenberg uncertainty for the particle in the $\hat{\mathbf{r}}_\mathbf{c}$ direction we have that: $\Delta x_\mathrm{e}\Delta p_\mathrm{e}\geq\hbar/2$ finally yielding Eq. 5 of the main text:

$$\Delta x_{\text{shw}} \Delta p_{\text{e}} \geq \frac{\hbar}{2}, \qquad \text{(S3.17)}$$

S4. Reconstruction of the wavefunction size

The photon density matrix is

$$\rho_{\text{ph}}(\omega', \omega) = \frac{\langle E^{(-)}(\omega') E^{(+)}(\omega) \rangle}{\int d\omega \, \langle E^{(-)}(\omega) E^{(+)}(\omega) \rangle}, \qquad \text{(S4.1)}$$

Explicitly it is dominated by three terms

$$\rho_{\text{ph}}(\omega', \omega) = \underbrace{T(\omega) T(\omega')}_{\text{detection}} \times \underbrace{\text{sinc}\left\{\frac{t_{\text{int}}}{2}\left[\omega - \omega' - (\omega + \omega')\frac{n(\omega)\omega - n(\omega')\omega'}{n(\omega)\omega - n(\omega')\omega'}\right]\right\}}_{\text{dispersion}}$$
$$\times \underbrace{\int d^3\mathbf{x}\, e^{i\frac{n(\omega)\omega - n(\omega')\omega'}{c}\hat{\mathbf{r}}_{\text{c}} \cdot \mathbf{x}} G_{\text{e}}^{(1)}(\mathbf{x}, \mathbf{x})}_{\text{wavefunction}}, \qquad \text{(S4.2)}$$

with $\text{Tr}\{\rho_{\text{ph}}\} = 1$. Let us approximate $G_{\text{e}}^{(1)}(\mathbf{x}, \mathbf{x})$ as a Gaussian probability cloud:

$$G_{\text{e}}^{(1)}(\mathbf{x}, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\det\{\boldsymbol{\Delta}\}} \exp\left(-\frac{1}{2}\mathbf{x}^T \boldsymbol{\Delta}^{-2} \mathbf{x}\right), \qquad \text{(S4.3)}$$

with $\int d^3\mathbf{x} G_{\text{e}}^{(1)}(\mathbf{x}, \mathbf{x}) = 1$. For this wavepacket shape we derive Eq. (7) of the main text.

Derivation:
The spatial Fourier transform of the probability cloud is

$$\int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} G_{\text{e}}^{(1)}(\mathbf{x}, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\det\{\boldsymbol{\Delta}\}} \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \exp\left(-\frac{1}{2}\mathbf{x}^T \boldsymbol{\Delta}^{-2}\mathbf{x}\right)$$
$$= \exp\left(-\frac{1}{2}\mathbf{q}^T \boldsymbol{\Delta}^2 \mathbf{q}\right), \qquad \text{(S4.4)}$$

Thus

$$\int d^3\mathbf{x}\, e^{i\frac{n(\omega)\omega - n(\omega')\omega'}{c}\hat{\mathbf{r}}_{\text{c}}\cdot\mathbf{x}} G_{\text{e}}^{(1)}(\mathbf{x}, \mathbf{x}) = \exp\left\{-\frac{1}{2}\hat{\mathbf{r}}_{\text{c}}^T \boldsymbol{\Delta}^2 \hat{\mathbf{r}}_{\text{c}}^T \left[\frac{n(\omega)\omega - n(\omega')\omega'}{c}\right]^2\right\},$$
$$\text{(S4.5)}$$

For weak material dispersion, the photon density matrix simplifies to

$$\rho_{\text{ph}}(\omega', \omega) \cong T(\omega) T(\omega') \exp\left[-\frac{(\omega - \omega')^2}{2(v_g^2/\hat{\mathbf{r}}_{\text{c}}^T \boldsymbol{\Delta}^2 \hat{\mathbf{r}}_{\text{c}}^T)}\right], \qquad \text{(S4.6)}$$

or, for a Gaussian detection,

$$\rho_{\text{ph}}(\omega', \omega)$$
$$\cong \frac{1}{\sqrt{2\pi}\Delta\omega_{\text{det}}} \exp\left[-\frac{\left(\frac{\omega + \omega'}{2} - \omega_0\right)^2}{2\Delta\omega_{\text{det}}^2}\right] \exp\left[-\frac{(\omega - \omega')^2}{8\Delta\omega_{\text{det}}^2}\right] \exp\left[-\frac{(\omega - \omega')^2}{2(v_g^2/\hat{\mathbf{r}}_{\text{c}}^T \boldsymbol{\Delta}^2 \hat{\mathbf{r}}_{\text{c}}^T)}\right],$$
$$\text{(S4.7)}$$

Define a rotated set of coordinates $\omega_+ = \frac{\omega + \omega'}{\sqrt{2}}, \omega_- = \frac{\omega - \omega'}{\sqrt{2}}$ then

$$\rho_{\text{ph}}(\omega_+, \omega_-)$$
$$\cong \frac{1}{\sqrt{2\pi}\Delta\omega_{\text{det}}} \exp\left[-\frac{\left(\frac{1}{\sqrt{2}}\omega_+ - \omega_0\right)^2}{2\Delta\omega_{\text{det}}^2}\right] \exp\left[-\frac{\omega_-^2}{4\Delta\omega_{\text{det}}^2}\right] \exp\left[-\frac{\omega_-^2}{(v_g^2/\hat{\mathbf{r}}_{\text{c}}^T \boldsymbol{\Delta}^2 \hat{\mathbf{r}}_{\text{c}}^T)}\right], \qquad \text{(S4.8)}$$

Then for the center of the bandwidth $\omega_+ = \sqrt{2}\omega_0$ we have

$$\rho_{\mathrm{ph}}(\omega_-) \cong \frac{1}{\sqrt{2\pi}\Delta\omega_{\mathrm{det}}} \exp\left[-\left(\frac{1}{4\Delta\omega_{\mathrm{det}}^2} + \frac{1}{v_g^2/\hat{\mathbf{r}}_{\mathrm{c}}^T \boldsymbol{\Delta}^2 \hat{\mathbf{r}}_{\mathrm{c}}^T}\right)\omega_-^2\right], \qquad \text{(S4.9)}$$

and ω_- denotes the coordinate on the off-diagonal direction, having a width $\Delta\omega_-$. If the detection bandwidth is broader than $\Delta\omega_-$, we find the relation

$$\hat{\mathbf{r}}_{\mathbf{c}}^T \boldsymbol{\Delta}^2 \hat{\mathbf{r}}_{\mathbf{c}} = \frac{v_g^2}{\Delta\omega_-^2}, \qquad \text{(S4.10)}$$

which is Eq. (7) of the main text.

<u>S5. A note on the case of finite and infinite interaction times</u>.

If t_{int} is finite, the sharpness of the delta function determining the Cherenkov will be degraded as well:

$$\begin{aligned}\delta[\omega - \beta c q(\omega)\cos\theta]\delta[\omega' - \beta c q(\omega')\cos\theta] & \\ \to \frac{1}{n\beta\omega}\frac{t_{\mathrm{int}} n\beta\omega}{2\pi}\operatorname{sinc}\left[\frac{t_{\mathrm{int}} n\beta\omega}{2}(\cos\theta - \cos\theta_{\mathrm{c}})\right] & \\ \times \frac{t_{\mathrm{int}}}{2\pi}\operatorname{sinc}\left[\frac{t_{\mathrm{int}}\left(1 - \frac{n_g}{n}\right)}{2}(\omega - \omega')\right], & \qquad \text{(S5.1)}\end{aligned}$$

The Cherenkov angle is well defined as long as $t_{\mathrm{int}} n\beta\omega \gg 1$. If the interaction time is strictly infinite, however, the second sinc function becomes a delta function of $\omega - \omega'$ and the coherence is dictated by the medium dispersion and no longer depends on the wavefunction. For a nondispersive medium $n_g = n$ this is not a problem. However, for weakly dispersive media, in order to ensure that the coherence is determined dominantly by the wavefunction, we wish to work in a regime where

$$t_{\mathrm{int}}\left(1 - \frac{n_g}{n}\right)\Delta\omega \ll 1, \qquad \text{(S5.2)}$$

For any relevant frequency interval $\Delta\omega$. Thus, the optimal range for t_{int} is

$$\frac{1}{n\beta\omega} \ll t_{\mathrm{int}} \ll \frac{1}{\left(1 - \frac{n_g}{n}\right)\Delta\omega}, \qquad \text{(S5.3)}$$

Or for the interaction length:

$$\frac{2\pi c}{n\omega} \ll L_{\mathrm{int}} \ll \frac{n}{n - n_g}\frac{2\pi v}{\Delta\omega}, \qquad \text{(S5.4)}$$

In terms of wavelength we get

$$\omega = \frac{2\pi c}{\lambda} \to \Delta\omega = \frac{2\pi c}{\lambda^2}\Delta\lambda, \qquad \text{(S5.5)}$$

$$\frac{\lambda}{n} \ll L_{\mathrm{int}} \ll \frac{n}{\Delta n}\frac{\lambda}{\Delta\lambda}\beta\lambda, \qquad \text{(S5.6)}$$

<u>S6. Conservation of particle momentum coherence in a general bulk medium</u>

Consider an initial electron density matrix $\rho_i(\mathbf{k}_i, \mathbf{k}_i')$ in the paraxial approximation with carrier velocity v_0, propagating in a in an arbitrary medium. We describe the medium by a dyadic Green function $\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)$, which can for example describe a lossy bulk, or other possible excitations that cause decoherence and loss of energy. The electron interacts with all the possible photonic quasi-particles excitable in the medium, including but not restricted to, plasmons, phonons and electron-hole pair excitations. Following one excitation in the weak-coupling regime the final electron state is prescribed by first-order time-dependent perturbation theory, with $H_{\text{int}} = \frac{e}{m}\mathbf{A}\cdot\mathbf{p}$, where the vector potential in such a medium reads [75]

$$\mathbf{A}(\mathbf{r}) = \sqrt{\frac{\hbar}{\pi\epsilon_0}\frac{1}{c^2}}\int \omega\, d\omega \int d^3\mathbf{r}'\sqrt{\operatorname{Im}\epsilon(\mathbf{r}',\omega)}\,\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\mathbf{f}(\mathbf{r}',\omega) + h.c., \qquad \text{(S6.1)}$$

where the vector operator $\mathbf{f}(\mathbf{r},\omega)$ annihilates an excitation at position $\mathbf{r}$ and frequency ω, with commutation relations $\left[f_\alpha(\mathbf{r},\omega), f_\beta^\dagger(\mathbf{r}',\omega')\right] = \delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')\delta_{\alpha\beta}$.

After the excitation, and since the excited quasiparticle is not measured, the electron final state is given as

$$\rho_f(\mathbf{k}_f,\mathbf{k}_f') = \sum_{\mathbf{k}_i,\mathbf{k}_i'} \rho_i(\mathbf{k}_i,\mathbf{k}_i') \sum_{\mathbf{r}\omega\alpha} M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{r}\omega\alpha} M^*_{\mathbf{k}_i'\to\mathbf{k}_f'\mathbf{r}\omega\alpha}, \qquad \text{(S6.2)}$$

where $M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{r}\omega\alpha}$ denotes the corresponding transition amplitude, given by:

$$M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{r}\omega\alpha} = \frac{i}{\hbar}\frac{e}{m}\int dt\,\langle \mathbf{k}_f;\mathbf{r}\omega\alpha|\mathbf{A}(\mathbf{r}')\cdot\mathbf{p}|\mathbf{k}_i;0\rangle, \qquad \text{(S6.3)}$$

Below we show how the *particle momentum coherence*, defined by the off diagonal parts of its density matrix

$$\varrho_{\mathrm{e}}(\mathbf{q}_\omega - \mathbf{q}_{\omega'}) = \int d^3\mathbf{k}\,\rho_{\mathrm{e}}(\mathbf{k}+\mathbf{q}_{\omega'},\mathbf{k}+\mathbf{q}_\omega)$$

is conserved in a spontaneous emission process (inelastic interactions) by a free electron interacting with an optical medium. Since the above quantity fully determines the coherence of the emitted light (Eq. (6) of the main text), the latter is retained even in the presence of other competing excitations.

Derivation:

We start by calculating the transition amplitude by substituting Eq. (S6.1) and by considering a long interaction $t\to\infty$, and the commutation relation of the $f_\alpha(\mathbf{r},\omega)$ operators, yielding:

$$\begin{aligned} M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{r}\omega\alpha} &= i\sqrt{\frac{\hbar}{\pi\epsilon_0}\frac{1}{c^2}}\,2\pi\delta\left(\frac{E_i-E_f}{\hbar}-\omega\right)\omega\sqrt{\operatorname{Im}\epsilon(\mathbf{r},\omega)} \\ &\times\int d^3\mathbf{r}'\,\mathbf{G}^*_{\gamma\alpha}(\mathbf{r}',\mathbf{r},\omega)\frac{e}{m}\mathbf{k}_{\gamma i}e^{i(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{r}'}, \qquad \text{(S6.4)}\end{aligned}$$

For the trace over the excitation we find:

$$\begin{aligned} &\sum_{\mathbf{r}\omega\alpha} M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{r}\omega\alpha} M^*_{\mathbf{k}_i'\to\mathbf{k}_f'\mathbf{r}\omega\alpha} \\ &\quad= \frac{e^2}{\pi\epsilon_0\hbar c^2}\,4\pi^2\int d\omega\,\delta\left(\frac{E_i-E_f}{\hbar}-\omega\right)\delta\left(\frac{E_i'-E_f'}{\hbar}-\omega\right) \\ &\quad\times\int d^3\mathbf{r}' e^{i(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{r}'}\int d^3\mathbf{r}'' e^{i\left(\mathbf{k}_i'-\mathbf{k}_f'\right)\cdot\mathbf{r}''} \\ &\quad\times\frac{\hbar\mathbf{k}_i}{m}\underbrace{\left[\int d^3\mathbf{r}\frac{\omega^2}{c^2}\operatorname{Im}\epsilon(\mathbf{r},\omega)\mathbf{G}^*(\mathbf{r}',\mathbf{r},\omega)\mathbf{G}(\mathbf{r}'',\mathbf{r},\omega)\right]}_{\operatorname{Im}\mathbf{G}(\mathbf{r}'',\mathbf{r}',\omega)}\frac{\hbar\mathbf{k}_i'}{m}, \qquad \text{(S6.5)}\end{aligned}$$

Thus, the most general relation between the initial and final density matrices is

$$\rho_f(\mathbf{k}_f,\mathbf{k}'_f)=\frac{1}{A}\int d^3\mathbf{k}_i d^3\mathbf{k}'_i\rho_i(\mathbf{k}_i,\mathbf{k}'_i)\delta(E'_i-E'_f-E_i+E_f)$$
$$\times\,\mathbf{v}_i\mathrm{Im}\,\mathbf{G}\left(\mathbf{k}'_i-\mathbf{k}'_f,\mathbf{k}_i-\mathbf{k}_f,\frac{E_i-E_f}{\hbar}\right)\mathbf{v}'_i\,,\qquad \text{(S6.6)}$$

where A is a normalization factor and $\mathbf{v}_i=\frac{\hbar\mathbf{k}_i}{m}$. Clearly, the final density matrix is given by the two-sided convolution with the response function $\mathrm{Im}\,\mathbf{G}(\mathbf{q},\mathbf{q}',\omega)$. A similar result in the spatial domain was derived by us in [57].

Now, let us approximate $v_0\cong\frac{\hbar k_i}{m}\hat{\mathbf{z}}\cong\frac{\hbar k'_i}{m}\hat{\mathbf{z}}$ and consider a homogeneous medium for which

$$\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)=\mathbf{G}(\mathbf{r}-\mathbf{r}',\omega)\rightarrow\mathbf{G}(\mathbf{q},\mathbf{q}',\omega)=(2\pi)^3\delta(\mathbf{q}-\mathbf{q}')\mathbf{G}(\mathbf{q},\omega),\qquad \text{(S6.7)}$$

and take the small-recoil approximation,

$$\frac{E_i-E_f}{\hbar}\cong(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{v}_0,\qquad \text{(S6.8)}$$

resulting in

$$\rho_f(\mathbf{k}_f,\mathbf{k}'_f)=\frac{1}{A}\int d^3\mathbf{k}_i d^3\mathbf{k}'_i\rho_i(\mathbf{k}_i,\mathbf{k}'_i)\delta(\mathbf{k}'_i-\mathbf{k}'_f-\mathbf{k}_i+\mathbf{k}_f)$$
$$\times\,\mathrm{Im}\,\mathbf{G}_{zz}\big(\mathbf{k}_i-\mathbf{k}_f,(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{v}_0\big)\,,\qquad \text{(S6.9)}$$

We now perform a change of coordinates, to illustrate the diagonal and off-diagonal dependence:

$$\bar{\mathbf{k}}_i=\frac{\mathbf{k}_i+\mathbf{k}'_i}{2},\qquad \Delta\mathbf{k}_i=\mathbf{k}_i-\mathbf{k}'_i,\qquad \text{(S6.11)}$$
$$\bar{\mathbf{k}}_f=\frac{\mathbf{k}_f+\mathbf{k}'_f}{2},\qquad \Delta\mathbf{k}_f=\mathbf{k}_f-\mathbf{k}'_f,\qquad \text{(S6.12)}$$

This yields

$$\rho_f(\bar{\mathbf{k}}_f,\Delta\mathbf{k}_f)=\frac{1}{A}\int d^3\bar{\mathbf{k}}_i d^3\Delta\mathbf{k}_i\rho_i(\bar{\mathbf{k}}_i,\Delta\mathbf{k}_i)\delta(\Delta\mathbf{k}_f-\Delta\mathbf{k}_i)\mathrm{Im}\,\mathbf{G}_{zz}\big(\bar{\mathbf{k}}_i-\bar{\mathbf{k}}_f,(\bar{\mathbf{k}}_i-\bar{\mathbf{k}}_f)\cdot\mathbf{v}_0\big)\,,\qquad \text{(S6.13)}$$

or, after integrating over $\Delta\mathbf{k}_i$, and redefining dummy integration variables,

$$\rho_f(\mathbf{k},\Delta\mathbf{k})=\frac{1}{\int d^3\mathbf{q}\,\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{q},\mathbf{q}\cdot\mathbf{v}_0)}\int d^3\mathbf{k}'\rho_i(\mathbf{k}',\Delta\mathbf{k})\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{k}-\mathbf{k}',(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}_0)\,,\qquad \text{(S6.14)}$$

where we have substituted A explicitly by computing the trace of the numerator (the trace of ρ_i is unity).

The results so far present the transformation of the density matrix in between scattering processes. Next, we note an important property of the transformation. Let us define two quantities: the particle *spectrum* (density matrix diagonal)

$$P(\mathbf{k})=\rho(\mathbf{k},0),\qquad \text{(S6.15)}$$

and the particle momentum coherence (which dictates the photon autocorrelations – see Eq. 6 in the text)

$$\varrho(\Delta\mathbf{k})=\int d^3\mathbf{k}\rho(\mathbf{k},\Delta\mathbf{k})\,,\qquad \text{(S6.16)}$$

From Eq. (S6.14) we see that

$$P_f(\mathbf{k})=\frac{1}{\int d^3\mathbf{q}\,\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{q},\mathbf{q}\cdot\mathbf{v}_0)}\int d^3\mathbf{k}'P_i(\mathbf{k}')\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{k}-\mathbf{k}',(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}_0)\,,\qquad \text{(S6.17)}$$

and

$$\varrho_f(\Delta\mathbf{k}) = \frac{1}{\int d^3\mathbf{q}\,\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{q},\mathbf{q}\cdot\mathbf{v}_0)}\int d^3\mathbf{k}\int d^3\mathbf{k}'\rho_i(\mathbf{k}',\Delta\mathbf{k})\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{k}-\mathbf{k}',(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}_0)$$
$$= \frac{1}{\int d^3\mathbf{q}\,\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{q},\mathbf{q}\cdot\mathbf{v}_0)}\int d^3\mathbf{q}\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{q},\mathbf{q}\cdot\mathbf{v}_0)\int d^3\mathbf{k}'\rho_i(\mathbf{k}',\Delta\mathbf{k})$$
$$= \int d^3\mathbf{k}'\rho_i(\mathbf{k}',\Delta\mathbf{k}) = \varrho_i(\Delta\mathbf{k}), \qquad \text{(S6.18)}$$

The final electron spectrum is a *convolution* of the initial spectrum with the material response $\mathrm{Im}\,\mathbf{G}_{zz}(\mathbf{q},\omega)$ – as expected from the standard theory of electron energy loss spectroscopy [85]. However, the particle momentum coherence is conserved:

$$\varrho_f(\Delta\mathbf{k}) = \varrho_i(\Delta\mathbf{k}), \qquad \text{(S6.19)}$$

meaning that the spectral autocorrelation features should persist even after the scattering events have taken place.

To summarize the important conclusion from this section: **the coherence of the particle is conserved during inelastic interactions,** as seen by the off-diagonal conservation of the density matrix. This conclusion is not contradicting the expectation that the classical uncertainty in the density matrix will increase during inelastic interactions. Indeed, the diagonal of the density matrix is increased by a convolution with the Green function of the medium that describes the excitations.

S7. **Comparison of classical, semiclassical and quantum shockwaves**

We now show how classical, semiclassical, and quantum theories give different results for the shockwave length and coherence. First, let us assume a model similar to the main text: Cherenkov radiation in a weakly dispersive medium, a detection system of finite bandwidth centred at optical frequencies (average group velocity v_g), far distanced observer at distance r and direction $\hat{\mathbf{r}}_{\mathrm{c}}$ (Cherenkov cone) relative to the emitting particle trajectory.

Both **classical** and **semiclassical** [1,34] theories compute the field amplitude from the relation

$$\mathbf{E}(\mathbf{r},\omega) = i\mu_0\omega \int d^3\mathbf{r}'\, \mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\mathbf{j}(\mathbf{r}',\omega), \qquad (S7.1)$$

where, for the **classical point particle** the current density is:

$$\mathbf{j}(\mathbf{r},t) = e\mathbf{v}_0\delta(\mathbf{r}-\mathbf{v}_0 t), \qquad (S7.2)$$

and for the **semiclassical particle** – the wavefunction is considered as a spread charge density [34]:

$$\mathbf{j}(\mathbf{r},t) = \mathbf{v}_0\rho(\mathbf{r}-\mathbf{v}_0 t) = e\mathbf{v}_0|\psi(\mathbf{r}-\mathbf{v}_0 t)|^2, \qquad (S7.3)$$

in effect, treating the wavefunction squared as a smeared-out charge density.

Going back to time domain and considering the model assumptions it is readily shown that the scalar field amplitude is

$$E(r,t) = \int d^3\mathbf{x}'\, |\psi(\mathbf{x}')|^2 \frac{\mathcal{A}_{\mathrm{p}}\left(r - v_g t - \hat{\mathbf{r}}_{\mathrm{c}}\cdot\mathbf{x}'\right)}{r}, \qquad (S7.4)$$

with $\mathcal{A}_{\mathrm{p}}\left(r - v_g t\right)$ being the field emitted by a point particle, i.e. when $|\psi(\mathbf{x})|^2 = \delta(\mathbf{x})$ is a classical point charge.

The **classical autocorrelations** are

$$\langle E(r,t')E(r,t)\rangle_{\mathrm{c}} = E(r,t')E(r,t) = \frac{\mathcal{A}_{\mathrm{p}}^*\left(r - v_g t'\right)}{r}\frac{\mathcal{A}_{\mathrm{p}}\left(r - v_g t\right)}{r}, \qquad (S7.5)$$

and the **classical instantaneous power** is

$$P_{\mathrm{c}}(r,t) = \langle E(r,t)E(r,t)\rangle_{\mathrm{c}} = \frac{\left|\mathcal{A}_{\mathrm{p}}\left(r - v_g t\right)\right|^2}{r^2}, \qquad (S7.6)$$

The **semiclassical autocorrelations** are

$$\langle E(r,t')E(r,t)\rangle_{\mathrm{sc}} = E(r,t')E(r,t)$$
$$= \int d^3\mathbf{x}'\, |\psi(\mathbf{x}')|^2 \frac{\mathcal{A}_{\mathrm{p}}^*\left(r - v_g t' - \hat{\mathbf{r}}_{\mathrm{c}}\cdot\mathbf{x}'\right)}{r} \int d^3\mathbf{x}\, |\psi(\mathbf{x})|^2 \frac{\mathcal{A}_{\mathrm{p}}\left(r - v_g t - \hat{\mathbf{r}}_{\mathrm{c}}\cdot\mathbf{x}\right)}{r}, \qquad (S7.7)$$

And the **semiclassical instantaneous power** is

$$P_{\mathrm{sc}}(r,t) = \langle E(r,t)E(r,t)\rangle_{\mathrm{sc}} = \left|\int d^3\mathbf{x}\, |\psi(\mathbf{x})|^2 \frac{\mathcal{A}_{\mathrm{p}}\left(r - v_g t - \hat{\mathbf{r}}_{\mathrm{c}}\cdot\mathbf{x}\right)}{r}\right|^2, \qquad (S7.8)$$

On the other hand, the **quantum autocorrelations** are given by Eq. (S1.19):

$$\langle E(r,t)E(r,t')\rangle_{\mathrm{q}} = \int d^3\mathbf{x}\, |\psi(\mathbf{x})|^2 \frac{\mathcal{A}_{\mathrm{p}}^*\left(r - v_g t' - \hat{\mathbf{r}}_{\mathrm{c}}\cdot\mathbf{x}\right)}{r}\frac{\mathcal{A}_{\mathrm{p}}\left(r - v_g t - \hat{\mathbf{r}}_{\mathrm{c}}\cdot\mathbf{x}\right)}{r}, \qquad (S7.9)$$

And the **quantum instantaneous power** is

$$P_{\rm q}(r,t) = \langle E(r,t)E(r,t)\rangle_{\rm q} = \int d^3\mathbf{x}\,|\psi(\mathbf{x})|^2 \frac{\left|\mathcal{A}_{\rm p}(r - v_g t - \hat{\mathbf{r}}_{\rm c}\cdot\mathbf{x})\right|^2}{r^2}, \qquad \text{(S7.10)}$$

In Figs. S1-S2 we plot numerical calculations of the field autocorrelations for the three theories. The parameters chosen correspond to spherical Gaussian pulses. The electron wavefunction size changes between $50nm$ and $1\mu m$, detection assumed to be centered around $\lambda = 600\text{nm}$ with $\Delta\lambda = 200\text{nm}$ bandwidth.

In the classical limit of wavefunction sizes much smaller than the emitted wavelength, both classical, semiclassical and quantum electrodynamics give the same result for the pulse duration and coherence. However, when the wavefunction is delocalized, the point-particle classical prediction is not affected, the semiclassical charge-density predicts a long pulse that is completely temporally coherent with itself, and quantum electrodynamics predicts an *even longer* pulse, only partially coherent – as seen by the narrow width of the off-diagonal terms of the field autocorrelation.

S8. **Momentum space derivation of Cherenkov autocorrelations**

Here we show how for Cherenkov radiation in a homogeneous medium, a momentum-space approach gives the same result as the Green function formalism – although it is much less general. Consider an emission process from an electron with initial state $|\psi\rangle = \sum_{\mathbf{k}} A_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\,|\mathbf{k}\rangle$ Let us consider a final bipartite state (here we add the transition matrix element, absorbed the time dependence into the kets and consider all final photon modes):

$$|\psi_f\rangle = \sum_{\mathbf{k}_i} A_{\mathbf{k}_i} \sum_{\mathbf{k}_f}\sum_{\mathbf{q}} M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{q}} |\mathbf{k}_f\rangle|\mathbf{q}\rangle, \qquad \text{(S8.1)}$$

where

$$M_{\mathbf{k}_i\to\mathbf{k}_f\mathbf{q}} = \frac{i}{\hbar}\frac{e}{m}\int dt\, e^{-i\left[\frac{E(\mathbf{k}_i)-E(\mathbf{k}_f)}{\hbar}-\omega\right]t} \langle \mathbf{k}_f\mathbf{q}|\mathbf{A}\cdot\mathbf{p}|\mathbf{k}_i;0\rangle, \qquad \text{(S8.2)}$$

is the transition matrix element. The final bipartite density matrix is $\rho_f = |\psi_f\rangle\langle\psi_f|$, and the observed photon density matrix $\rho_{\rm ph} = \mathrm{Tr}_{\rm el}\{\rho_f\}$ - for the simple case of Cherenkov radiation (where momentum is strictly conserved) - is

$$\rho_{\rm ph} \propto \sum_{\mathbf{q}}\sum_{\mathbf{q}'}\sum_{\mathbf{k}_f} A_{\mathbf{k}_f+\mathbf{q}} A^*_{\mathbf{k}_f+\mathbf{q}'}\,|\mathbf{q}\rangle\langle\mathbf{q}'|, \qquad \text{(S8.3)}$$

The probability of detecting a photon at $\mathbf{q}$:

$$p(\mathbf{q}) = \rho_{\rm ph}(\mathbf{q},\mathbf{q}) \propto \sum_{\mathbf{k}_f} \left|A_{\mathbf{k}_f+\mathbf{q}}\right|^2 = \sum_{\mathbf{k}_f}\left|A_{\mathbf{k}_f}\right|^2 = \text{electron independent}, \qquad \text{(S8.4)}$$

Confirming that observables such as the power spectrum are indeed wavefunction-independent. In contrast, in the photon autocorrelations (*after* the electron is traced out), we get information about the wavefunction – the momentum-space equivalent of Eq. 4 in the the manuscript:

$$\langle E(\mathbf{q})E(\mathbf{q}')\rangle \propto \rho_{\rm ph}(\mathbf{q},\mathbf{q}') \propto \sum_{\mathbf{k}_f} A_{\mathbf{k}_f+\mathbf{q}} A^*_{\mathbf{k}_f+\mathbf{q}'} = \int d^3\mathbf{r}\, e^{-i(\mathbf{q}'-\mathbf{q})\cdot\mathbf{r}} |\psi(\mathbf{r})|^2, \qquad \text{(S8.5)}$$

where $\psi(\mathbf{r}) = \sum_{\mathbf{k}} A_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$ is the initial electron wavefunction. Finally, let us compare this approach with the Green-function formalism that we developed earlier. The above momentum-space derivation is simple, yet it only *strictly* holds for Cherenkov radiation in homogeneous lossless media, and not to *all* coherent CL processes. The Green-function formalism holds for *all types* of excitations and media, and even covers the case of multiple electron emitters, since it employs electron second quantization. Thus, our formalism enables to directly generalize the conclusions of this work regarding the simple test-case of Cherenkov radiation to other emission mechanisms such as Smith-Purcell and transition radiation, once the Green function is prescribed.

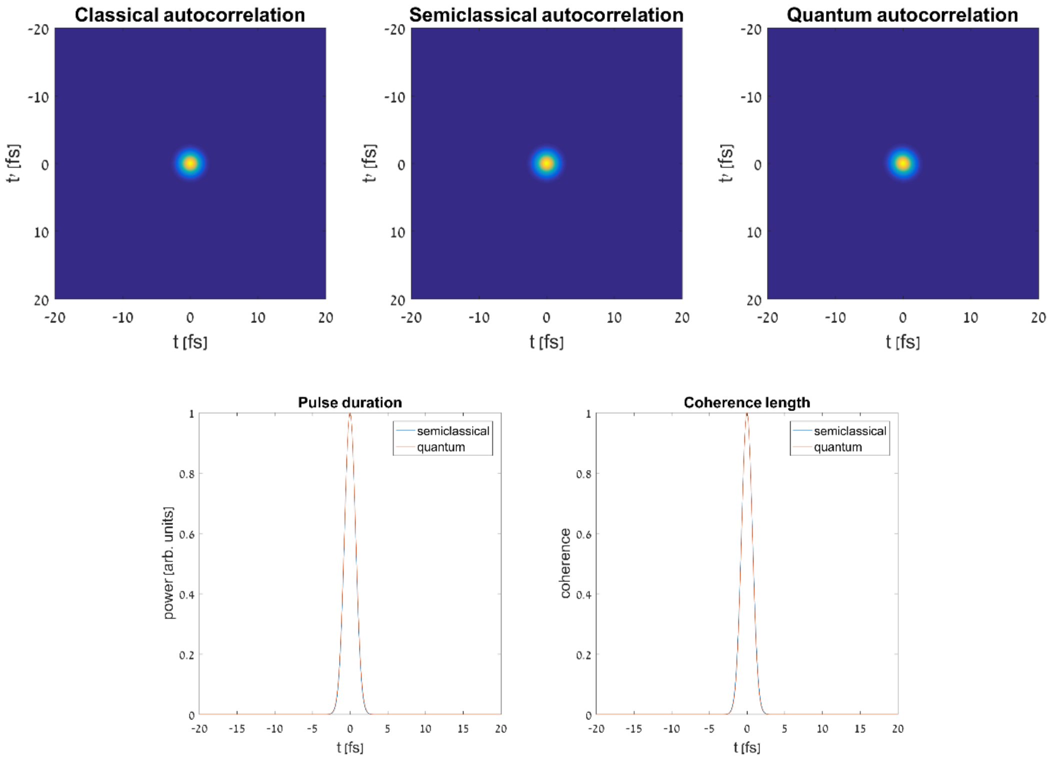

Fig. S1: Classical, semiclassical and quantum temporal autocorrelations of a Cherenkov pulse, for a detection system centered around $\lambda = 600\ nm$ and of bandwidth $\Delta\lambda = 200\ nm$. The wavefunction size of the electron is $50nm$ corresponding to a coherent energy uncertainty $\Delta E = 3.7eV$. Lower panel: cross-sections of the electron autocorrelations. The instantaneous pulse duration is given by the diagonal of the autocorrelations, and the temporal coherence is given by off-diagonal cross-section.

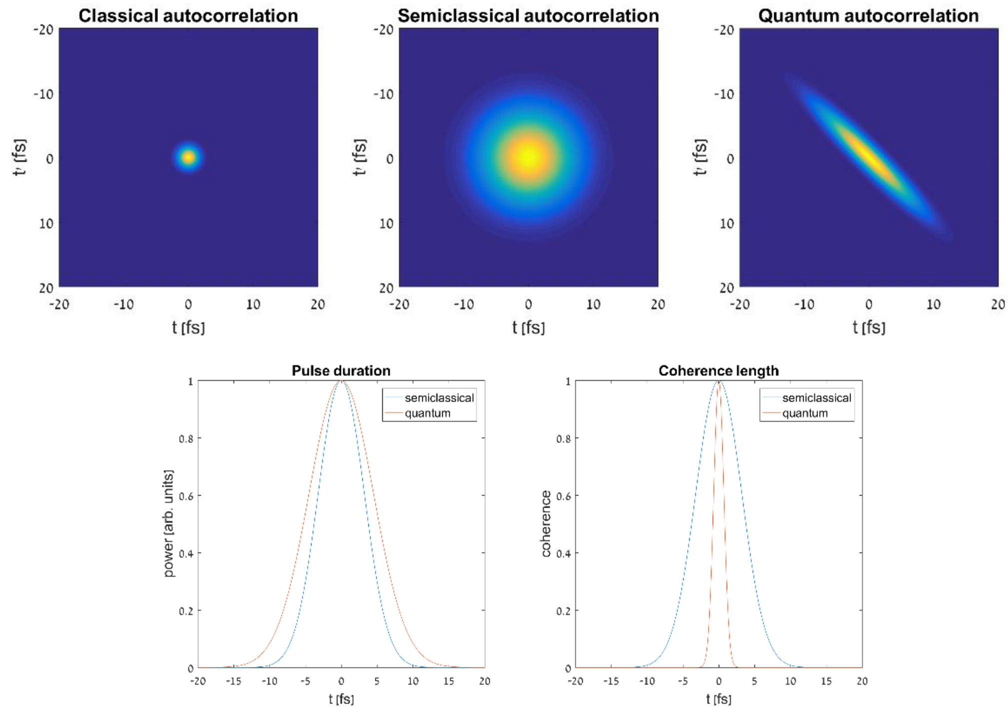

Fig. S2: Classical, semiclassical and quantum temporal autocorrelations of a Cherenkov pulse, for a detection system centered around $\lambda = 600\ nm$ and of bandwidth $\Delta\lambda = 200\ nm$. The wavefunction size of the electron is $1\mu m$ corresponding to a coherent energy uncertainty $\Delta E = 0.2eV$. Lower panel: cross-sections of the electron autocorrelations. The instantaneous pulse duration is given by the diagonal of the autocorrelations, and the temporal coherence is given by off-diagonal cross-section.

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